

Exercise Package 6:

Fourier series:

1. For the following pairs of signal $x(t)$ and $f(t)$ for $0 \leq t \leq 10$, sketch the functions and find the coefficient c such that $x(t) = c \cdot f(t)$ with minimum error energy (also calculate the energy of error $e(t) = x(t) - c \cdot f(t)$).
 - a. $x(t) = 0.25t[u(t) - u(t-10)]$ and $f(t) = -[u(t) - u(t-5)] + [u(t-5) - u(t-10)]$
 - b. $x(t) = 0.25t[u(t) - u(t-10)]$ and $f(t) = -0.2[u(t) - u(t-4)] + [u(t-4) - u(t-10)]$
 - c. $x(t) = e^{-0.25t}[u(t) - u(t-10)]$ and $f(t) = u(t) - u(t-4)$
 - d. $x(t) = e^{-0.25t}[u(t) - u(t-10)]$ and $f(t) = u(t) - u(t-5)$
 - e. $x(t) = 2[u(t) - u(t-4)] - [u(t-4) - u(t-10)]$ and $f(t) = -2[u(t) - u(t-4)] + [u(t-4) - u(t-10)]$
2. $x(t)$ is a 10KHz, 40% duty cycle, 3.3V square wave.
 - a. Find the power of $x(t)$.
 - b. Find Fourier series coefficients of $x(t)$ and sketch the line spectrum C_n , with $n = -3$ to 3 for magnitude and phase. (Tip: check out the Matlab function **stem(n, Cn)**)
 - c. Find the Fourier series expression of $x(t)$ using only the coefficients C_n , with $n = -3$ to 3 in real valued cosine function format. And calculate the power of the expression using the coefficient C_n .
 - d. Find the power of error in the expression of part c. (Tip: $P_{\text{error}} = (P_{\text{expression}} - P_{\text{signal}}) / P_{\text{signal}}$)
 - e. How many Fourier series coefficients are needed to express the signal $x(t)$ with the power of error less than 1% of the signal power? Show the verification of error.
3. $y(t)$ is a 1.5KHz, 70% duty cycle, 40V square wave.
 - a. Find the power of $y(t)$.
 - b. Find Fourier series coefficients of $y(t)$.
 - c. Find the Fourier series expression of $y(t)$ in real valued cosine function format such that the power of error is less than 10% of the power of $y(t)$. Show the verification of error.
4. Find the fundamental frequency and Fourier series coefficient C_n of the following signals and sketch the magnitude and phase line spectra of the signals: x
 - a. $x(t) = 2 + \cos(20\pi t + \pi/3) - 2\cos(50\pi t) + 4\sin(70\pi t + \pi/3)$
 - b. $x(t) = 2 + \cos(200\pi t + \pi/3) - 2\cos(50\pi t) + 4\sin(150\pi t + \pi/3)$
 - c. $x(t) = 2 + \cos(40\pi t + \pi/3) - 2\cos(5t) + 4\sin(10\pi t + \pi/3)$
 - d. $x(t) = 2 + \cos(20t + \pi/3) - 2\cos(50t) + 4\sin(70t + \pi/3)$
5. Given the following sets of Fourier series coefficients of real valued signals, find the expression of signal $x(t)$ in time domain:
 - a. Fundamental frequency $\omega_0 = 10$ rad/second, $C_0 = 2$, $C_2 = 2j$, and $C_{-5} = 1 + j$
 - b. Fundamental frequency $\omega_0 = 120\pi$ rad/second, $C_{-1} = 2$, $C_3 = 0.5e^{-j}$, and $C_{-5} = 0.3 + 0.1j$
6. Given a full-wave rectified signal $x(t) = |167\cos(120\pi t)|$ (V):
 - a. Calculate the average power of the signal $x(t)$.
 - b. Find its Fourier series coefficients and sketch the magnitude and phase line spectra.
 - c. Show that DC components C_0 is equal to $0.636 \cdot 167$ V.
 - d. Find the Fourier series expression of $x(t)$ in real valued cosine function format such that the power of error is less than 5% of the power of $x(t)$. Show the verification of error.

7. Given a half-wave rectified signal $x(t) = \begin{cases} |16\sin(100\pi t)| & \text{for } 0 \leq t \leq \frac{T_0}{2} \\ 0 & \text{for } \frac{T_0}{2} \leq t \leq T_0 \end{cases}$ (where $T_0 = 2\pi/\omega_0$

which is 100π):

- Calculate the average power P_x of the signal $x(t)$.
- Find its Fourier series coefficients and sketch the magnitude and phase line spectra.
- Show that DC components C_0 is equal to $0.318 \cdot 16V$.
- According to the Parseval's theorem, $P_x = \sum_{-\infty}^{\infty} |C_n|^2$, if only the first 11 coefficients ($n = -5, -4, \dots, -1, 0, 1, \dots, 4, 5$) are used in the expression, what is the error of power for this representation. Show the verification of error.