

Math 124 - Trigonometry

Exam 4

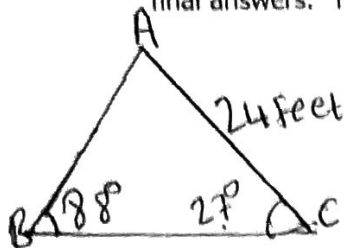
(McKeague 8th Ed. - Chapter 7 & 2.5. B.1. B.2)

Name YASSER AMER
Form #

1. (8 points) Find all the missing parts of triangle ABC if

$$B = 88^\circ, C = 27^\circ, \text{ and } b = 24 \text{ feet.}$$

Round to the nearest tenth of a foot and use correct units in your final answers. Then find the area of triangle ABC.



SINE RULE

$$\frac{c}{\sin 27^\circ} = \frac{24}{\sin 88^\circ}$$

$$c \sin 88^\circ = 24 \sin 27^\circ$$

$$c = \frac{24 \sin 27^\circ}{\sin 88^\circ}$$

$$AB = c = 10.90 \text{ feet}$$

$$\angle BAC = 180^\circ - (88^\circ + 27^\circ) = 65^\circ$$

$$\frac{a}{\sin 65^\circ} = \frac{24}{\sin 88^\circ}$$

$$a \sin 88^\circ = 24 \sin 65^\circ$$

$$a = \frac{24 \sin 65^\circ}{\sin 88^\circ} = 21.76$$

AREA

$$\frac{1}{2} bc \sin A$$

$$\frac{1}{2} \times 21.76 \times 24 \times \sin 27^\circ$$

$$= 0.5 \times 522.24 \times \sin 27^\circ$$

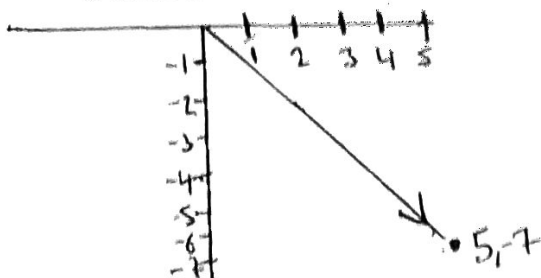
$$= 261.12 \sin 27^\circ$$

$$= 118.5459993$$

Area: 118.5459993
Square Units

2. $u = 5i - 7j$

2. (2 points) Draw vector $U = (5, -7)$. Then write U in vector component form in terms of the unit vectors i and j .



$$u = 5i - 7j$$

3. (3 points) Find the magnitude of the horizontal and vertical components for the vector V , given that $|V| = 15$ and $\theta = 223^\circ$. Round to one decimal place.

$$|V| = 15, \theta = 223^\circ$$

$$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$$

Horizontal Component

$$|V_x| = |V| \cos \theta$$

Vertical Component

$$|V_y| = |V| \sin \theta$$

$$V_x = 15 \cos 223^\circ = -10.97 \approx -11$$

$$V_y = 15 \sin 223^\circ = -10.23 \approx -10.2$$

$$|V_x| = 11$$

$$|V_y| = 10.23 \approx 10.2$$

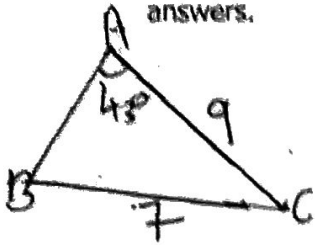
3. $|V_x| = 11$ Units

$|V_y| = 10.2$ Units

4. (10 points) Find all possible solutions for triangle ABC if

$$A = 43^\circ, a = 7 \text{ in}, b = 9 \text{ in}$$

Round to the nearest tenth and use correct units in your final answers.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$9 \sin 43^\circ = 7 \sin B$$

$$B = \sin^{-1} \left[\frac{9 \sin 43^\circ}{7} \right]$$

$$= 61.27^\circ$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$C = \frac{7 \sin 75.73^\circ}{\sin 43^\circ} = 9.95$$

$$\angle ACB = 180^\circ - (43^\circ + 61.27^\circ)$$

$$\angle ACB = 75.73^\circ$$

$$AB = 9.95 \text{ in}$$

AREA

$$\frac{1}{2} ab \sin A$$

$$A = \frac{1}{2} \times 9.95 \times 7 \times \sin 43^\circ$$

$$= \frac{1}{2} \times 89.55 \sin 43^\circ$$

$$= 30.54 \text{ square units}$$

$$C = 75.73^\circ$$

$$B = 61.27^\circ$$

$$AB = 9.95 \text{ in}$$

$$\text{Area} \rightarrow A = 30.54 \text{ square units}$$

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$$\text{Area} \rightarrow 30.52 \text{ square units}$$

$$\text{or } A = \frac{1}{2} \times ab \sin C$$

$$= \frac{1}{2} \times 7 \times 9.95 \sin 75.73^\circ$$

$$= 30.53 \text{ square units}$$

$$\text{or } A = \frac{1}{2} \times bc \sin B$$

$$= \frac{1}{2} \times 7 \times 9.95 \sin 61.27^\circ$$

$$= 44.775 \sin 61.27^\circ = 30.54$$

5. (8 points) Suppose triangle ABC has sides lengths $a = 6, b = 11$, and $c = 12$.

5. Area: 32.8 sq. units

- a. Find the area using Heron's formula. Round your final answer to one decimal place.
- b. Find the measure of each angle in triangle ABC. Round each angle to the nearest tenth of a degree.

$$A = 29.84^\circ$$

$$B = 65.81^\circ$$

$$C = 84.35^\circ$$

$$s = \frac{a+b+c}{2} = \frac{6+11+12}{2} = 14.5$$

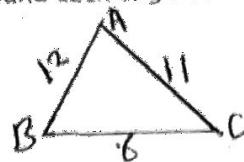
$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{14.5(14.5-6)(14.5-11)(14.5-12)}$$

$$= \sqrt{14.5(8.5)(3.5)(2.5)}$$

$$= \sqrt{1078.4375}$$

$$\approx 32.8396 \approx 32.8 \text{ square units}$$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{11^2 + 12^2 - 6^2}{2 \times 11 \times 12} = 0.1867$$

$$A = \cos^{-1}(0.1867) = 29.84^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = 0.409722$$

$$B = \cos^{-1} 0.409722 = 65.81^\circ$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = 0.09848$$

$$C = \cos^{-1} 0.09848 = 84.35^\circ$$

6. Consider the vectors:

$$U = \langle -6, -8 \rangle, V = \langle 2, 5 \rangle, W = \langle -1, 9 \rangle, \text{ and } X = \langle 4, -3 \rangle$$

a. (2 points) Find $X \cdot V$.

$$X \cdot V = (4i - 3j) \cdot (2, 5)$$

$$= (4 \times 2) + (-3 \times 5)$$

$$8 - 15 = -7$$

b. (2 points) Find $U - X$.

$$U - X = \langle -6, -8 \rangle - \langle 4i - 3j \rangle$$

$$= (-6 - 4)i + (-8 + 3j)$$

$$= -10i - 5j$$

c. (3 points) Find $3W - 2V$.

$$3W - 2V = 3\langle -1 + 9j \rangle - 2\langle 2, 5 \rangle$$

$$= \langle -3 + 27j \rangle - \langle 4, 10 \rangle$$

$$= \langle -7i + 17j \rangle$$

d. (4 points) Find angle θ between vectors V and X . Round to the nearest tenth of a degree.

$$V = \langle 2, 5 \rangle$$

$$X = 4i - 3j$$

$$V \cdot X = (2, 5) \cdot (4i - 3j)$$

$$= 8 + (-15) = -7$$

$$|V| = \sqrt{2^2 + 5^2}$$

$$|V| = \sqrt{29}$$

$$|X| = \sqrt{4^2 + 3^2}$$

$$= 5$$

$$\cos \theta = \frac{V \cdot X}{\|V\| \|X\|} = \frac{-7}{5\sqrt{29}}$$

$$\theta = \cos^{-1}(-0.259)$$

$$d. 105.07^\circ$$

e. (2 points) Are any of these vectors perpendicular? If so, state which pair(s) are perpendicular and explain your reasoning.

U and V are perpendicular
If $U \cdot V = 0$

$$U = \langle -6, -8 \rangle$$

$$X = 4i - 3j$$

$$(U \cdot X)$$

$$\langle -6, -8 \rangle \cdot \langle 4i - 3j \rangle$$

$$= -24 + 24 = 0 \Rightarrow \text{perpendicular}$$

e. U and X

7. (2 points each) Consider the complex numbers $z_1 = 4 - 6i$ and $z_2 = -3 + 4i$.

a. $z_1 - z_2 = (4 - 6i) - (-3 + 4i) = 7 - 10i$

7a. $7 - 10i$

b. $\frac{z_2}{z_1} = \frac{-3 + 4i}{4 - 6i} \times \frac{4 + 6i}{4 + 6i} = \frac{-12 - 18i + 16i + 24i^2}{16 + 36} = \frac{-36 - 2i}{52} = -\frac{9}{13} - \frac{1}{26}i$

b. $-\frac{9}{13} - \frac{1}{26}i$

$$= -\frac{9}{13} - \frac{1}{26}i$$

c. Convert z_1 into its equivalent trigonometric form. You may use "r cis θ " form.

$$Z = r(\cos \theta + i \sin \theta)$$

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$$

$$\tan \theta = \frac{y}{x} = \frac{6}{4} = \frac{3}{2}$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$= 56.31^\circ$$

$$Z = 2\sqrt{13}(\cos 56.31^\circ + i \sin 56.31^\circ)$$

ANS