

Assignment 1

Due time: 4:00pm, 8 April, 2019

Problem 1 (10 marks). Answer the following questions and briefly explain your reasoning:

- a) What do skewness and kurtosis of a variable imply for its distribution? [5 marks]
- b) What is the difference between autocorrelation and partial autocorrelation functions?
How can we use these two statistics to identify the autoregressive and moving average components of a time series? [5 marks]

Problem 2 (20 marks). Determine which of the following processes are stationary and invertible. In the following we always assume that u_t is white noise with mean zero and variance σ^2 , i.e. $u_t \sim \text{WN}(0, \sigma^2)$.

- a) $y_t = -0.2y_{t-1} + 0.48y_{t-2} + u_t$; [5 marks]
- b) $y_t = -1.9y_{t-1} - 0.88y_{t-2} + u_t + 1.8u_{t-1} + 0.81u_{t-2}$; [5 marks]
- c) $y_t + 1.6y_{t-1} = u_t - 0.4u_{t-1} + 0.04u_{t-2}$; [5 marks]
- d) $y_t = u_t - \frac{10}{3}u_{t-2}$. [5 marks]

Problem 3 (10 marks). Consider the following ARMA(2,4) process, with α, β as real parameters, i.e. $\alpha, \beta \in \mathbb{R}$,

$$y_t = \alpha y_{t-2} + u_t + \frac{1}{\beta} u_{t-4}, \quad u_t \sim \text{WN}(0, 1)$$

State the range of values for α and β for which y_t is stationary and invertible?

Problem 4 (30 marks). a) Inflation, denoted as $\{X_t\}$, is a key target variable for monetary policy and if its forecasts are outside the interval $[0.5, 3]$, then we say that predicted inflation is off-target and the Central Bank needs to intervene.

- (i) The last $T = 100$ values of $\{X_t\}$ give the following sample autocorrelations $\hat{\rho}(k)$ and partial autocorrelations $\hat{\pi}(k)$:

lag	k=1	k=2	k=3	k=4
$\hat{\rho}(k)$	0.9144	0.7620	0.5984	0.4500
$\hat{\pi}(k)$	0.9144	-0.4523	0.0883	-0.0258

The sample mean is $\bar{X} = 2.0593$ and sample variance is $\hat{\sigma}_x^2 = 2.0658$. Using an approximate formula for the 95% confidence intervals for $\hat{\rho}(k)$ and $\hat{\pi}(k)$ explain why an AR(2) seems to be a reasonable model for $\{X_t\}$. [6 marks]

(ii) Write an AR(2) model for $\{X_t\}$ and estimate all its parameters. [12 marks]

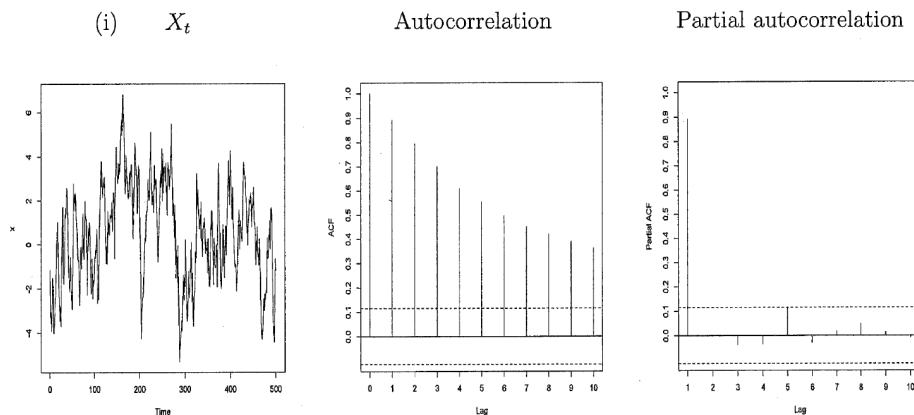
b) The sample used in part a) contains the recent Financial Crisis, therefore, in order to avoid extreme values, a shorter sample is considered. In particular, for the last $T = 49$ values of $\{X_t\}$, we assume to have the same sample autocorrelations $\hat{\rho}(k)$ and partial autocorrelations $\hat{\pi}(k)$ and the same sample mean, $\bar{X} = 2.0593$, as in part (a.i), but the sample variance is now reduced to $\hat{\sigma}_x^2 = 0.6251$. Consider again an AR(2) model for inflation and estimate again only those parameters that have changed. [6 marks]

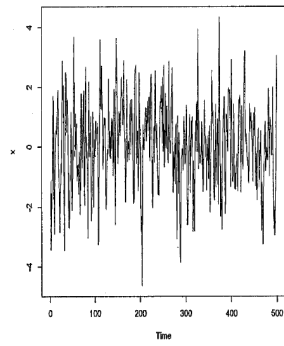
c) Give one reason for preferring the approach in part (a) and one reason for preferring the approach in part (b). [6 marks]

Problem 5 (30 marks). In each of the following three sets of graphs is shown a realisation of the process

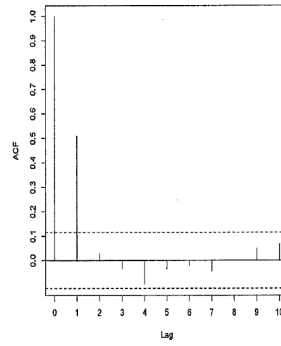
$$X_t = \phi X_{t-1} + u_t + \theta u_{t-1}, \quad u_t \sim \text{WN}(0, 1)$$

together with its estimated autocorrelation and partial autocorrelation functions and their 99% confidence intervals. Give values for ϕ and θ in all cases and state if the process is stationary and invertible.

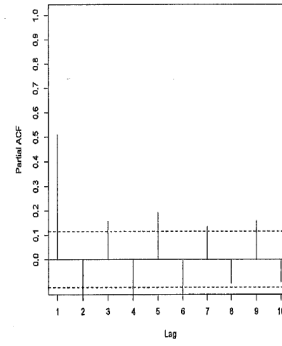
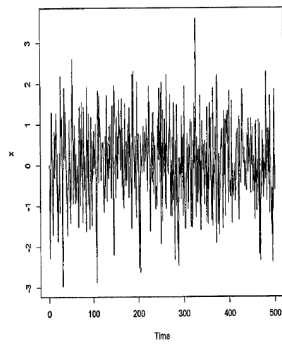


(ii) X_t 

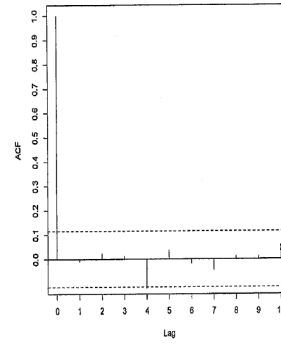
Autocorrelation



Partial autocorrelation

(iii) X_t 

Autocorrelation



Partial autocorrelation

