

Assignment 2

Due time: 4:00pm, 9 October, 2020 (New Zealand Time)

Problem 1 (35 marks). Consider the following ARMA(1,1) model for some time series data

$$y_t = 0.036 + 0.69y_{t-1} + 0.42u_{t-1} + u_t$$

Suppose that you have data for time to $t-1$, i.e. you know that $y_{t-1} = 3.4$, and $u_{t-1} = -1.3$

- a) Obtain forecasts for the series y for times t , $t+1$, and $t+2$ using the estimated ARMA model. [8 marks]
- b) If the actual values for the series turned out to be -0.032, 0.961, 0.203 for t , $t+1$, $t+2$, calculate the (out-of-sample) mean squared error, mean absolute error and percentage of correct sign predictions. Comment on the differing characteristics of the three methods used for evaluating the forecasting accuracy. [12 marks]
- c) What procedure might be used to estimate the parameters of an ARMA model? Explain, briefly, how such a procedure operates, and why OLS is not appropriate. [5 marks]
- d) Suppose that we find ARCH effects in the data and use a GARCH model to capture the dependence structure in the variance. What procedure might be used to estimate the parameters of a GARCH model? Explain, briefly, how such a procedure operates, and why OLS is not appropriate. Also comment on the possible problems of the procedure you suggest. [10 marks]

Problem 2 (25 marks). Suppose that we have an ARCH(2) model for financial returns

$$\begin{aligned} u_t &= \epsilon_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 \quad \epsilon_t \sim \text{iid } \mathcal{N}(0, 1) \end{aligned}$$

where ϵ_t is independent of σ_t , $\alpha_1 \geq 0, \alpha_2 \geq 0$ and $\alpha_0 > 0$. We assume that $\{u_t\}$ is weakly stationary.

- a) Derive the unconditional mean and variance of $\{u_t\}$. [10 marks]
- b) Derive a condition on the parameters which ensures weak stationarity of the $\{u_t\}$ process. [5 marks]

- c) Suppose that we have estimated the ARCH(2) model and obtained the parameter estimates: $\hat{\alpha}_0 = 1$, $\hat{\alpha}_1 = 0.5$, and $\hat{\alpha}_2 = 0.1$. Assume that $u_T^2 = 1$, and $u_{T-1}^2 = 0$ are observed. Denote by $\sigma_{T+h|T}^2$ the h-step-ahead forecast of the conditional variance. Compute the one- and two-step-ahead forecasts of the conditional variance σ_t^2 .

[10 marks]

Problem 3 (40 marks). The basic GARCH(1,1) model is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (1)$$

where σ_t^2 is the conditional variance of the error u_t in the mean equation of the time series model for $\{y_t\}$.

- a) Explain the intuition underlying equation (1). [6 marks]
- b) Explain why GARCH models are likely to be more useful models than ARMA type models for modelling time series of financial returns. [6 marks]
- c) Explain why, in recent empirical research, researchers prefer GARCH(1,1) models to pure ARCH(q) models? [8 marks]
- d) The mean equation for $\{y_t\}$ is given by

$$y_t = \mu + u_t$$

where μ is a constant. If y_t is a daily stock return series, what range of values are likely for the coefficients μ, α_0, α_1 and β_1 ? [6 marks]

- e) Suppose now that the researcher had estimated the above GARCH model for a series of returns on a stock index and obtained the following parameter estimates: $\hat{\mu} = 0.0023, \hat{\alpha}_0 = 0.0172, \hat{\alpha}_1 = 0.1251, \hat{\beta}_1 = 0.8611$. If the researcher has data available up to and including time T, write down a set of equations in σ_T^2 and u_T^2 and their lagged values, which could be employed to produce one-, two-, and three-step-ahead forecasts for the conditional variance of u_t . Derive equations of the one-, two-, and three-step-ahead forecasts for σ_T^2 . [14 marks]