Investments - FIN460 - Spring 2019
Professor Christopher Hrdlicka
Final Exam

Directions

- This exam is closed book. You may use a calculator but no cell phones, laptops or other electronic devices may be used.
- The formula sheet is located at the end of the exam. You may detach those pages. Do not separate any of the remaining exam.
- There is extra space at the end of the exam for use in calculation. Show your final work with the actual question or indicate where it can be found.
- Write your name on both cover sheets and the upper right hand corner of each page. Separated pages without a name *cannot* be graded.
- You must show your work or provide an explanation to receive credit. e.g. A number without work or an answer of True or False without explanation will receive no credit even if correct.
- Write your answers in the space provided. Make clear what you want graded. Circle your final numerical answer and cross-out completely work you do not want graded. If you have conflicting answers or work and we cannot easily tell what your final answer is *no* partial credit can be awarded. If you write your answer on the back of the page you must indicate so or it cannot be graded.
- You have 100 minutes to complete the exam. The point values of each question are indicated and can serve as a rough suggestion of how much time in minutes to spend on each problem.
- This exam has 22 pages including the scratch paper, formula sheet, etc. The exam has 10 questions. Check your exam to see that you indeed have a complete exam.

Name: ____________________

The work on this exam must be entirely your own.

Investments - FIN460 - Spring 2019
Professor Christopher Hrdlicka
Final Exam

Name: ______________________

Exam Score (Max 100): ______________________

True/False: 20 Points Total

Directions: Answer each question by writing either "True" or "False" followed by a short explanation of approximately a few well constructed sentences. Answers without explanation will results in no credit.

1. (5 Points) Bonds with higher yields to maturity have higher expected returns.

2. (5 points) Very risk averse investors should invest in safe stocks while less risk averse investors should invest in riskier stocks.

3. (5 points) If the CAPM holds (and you only have financial wealth) you should just invest in the market, e.g., the Vanguard Total Market Index, mixed with bonds.

4. (5 points) There is nothing special we need to do to adjust our asset pricing models for hedge funds holding illiquid assets.

Short Answer: 30 Points Total

Directions: Answer each question with a short answer explaining your reasoning. Each answer should consist of a few well constructed sentences. Answers such as “Yes” or “No” without explanation will result in no credit.

1. (10 Points) Define the Sharpe Ratio. Explain when and why you can and cannot use the Sharpe ratio as good measure of the risk return trade off.

2. (10 Points) Explain how matching assets and liabilities on both their duration and convexity help eliminate more interest rate risk than matching on duration alone?

3. (10 Points) Alpha: How do we measure it? What is it? How do we use it (give two distinct interpretations of it)? And explain how these two interpretation highlight a fundamental conflict in using alpha that we discussed throughout the course.

Quantitative: 50 Points Total

1. (10 Points) You know the following

$$E[r^e_{mkt}] = 4\%, \quad E[r_{SMB}] = 2\%, \quad E[r_{HML}] = 3\%, \quad r_f = 1\%$$

You can only take long positions. You believe this data is representative of the future returns.

   (a) (4 Points) You see a stock with a CAPM market beta of 1.25. The stock has an expected return of 7%. If you believe the CAPM is the true model of the asset returns, is this stock a good investment and why or why not?

   (b) (6 Points) Your investment adviser says the Fama French 3 factor model is a better model. The stock has the following betas: market beta of 0.8, smb beta of -0.1, and hml beta of 0.9. If you believe your investment adviser, is the stock a good investment.

Extra Space:

2. (15 Points Total) You are working in Apple's corportate treasury. You have been given a portfolio of $150 million to manage. The CFO tells you that he wants to have half of this capital available in 3 years to invest in new start ups and the other half available in 7 years, i.e., you have a 3 year and 7 year liability each with a present value of $75 million.

   You look out at financial markets and you have a 1 year, 5 year and 10 year zero coupon default free bonds available. Interest rates are flat at 5%

   (a) (5 Points) How should you invest this money to eliminate the risk of interest rates all rising or falling together? Tell me how much you would invest in each bond.

   (b) (10 Points) How should you invest this money to eliminate as much risk in interest rate moves as possible. (Hint: Use all three bonds.)

Extra Space:

3. (25 Points Total) There are three risky assets in which you can invest. They have the following mean, variance-covariance matrix and inverse matrix:

$$
\begin{array}{c}
 \\ A \\ B \\ C
\end{array}
\begin{array}{c}
\mu \\
\begin{pmatrix} 0.05 \\ 0.08 \\ 0.1 \end{pmatrix}
\end{array}
\begin{array}{c}
\Sigma \\
\begin{pmatrix} 0.01 & 0.01 & 0.015 \\ 0.01 & 0.04 & 0.03 \\ 0.015 & 0.03 & 0.09 \end{pmatrix}
\end{array}
\begin{array}{c}
\Sigma^{-1} \\
\begin{pmatrix} 150. & -25. & -16.6667 \\ -25. & 37.5 & -8.33333 \\ -16.6667 & -8.33333 & 16.6667 \end{pmatrix}
\end{array}
$$

The risk free rate of return is 0.01. You have no access to leverage on your own–so you cannot move beyond the tangency portfolio on the capital allocation line. You consider investing in a hedge fund that can provide access to leverage and thereby allow you to move out further on the capital allocation line. The hedge fund invests in the tangency portfolio leveraged 100%. Let's calculate how much better off the hedge fund makes you by allowing you to do this.

(a) (5 Points) What are the weights in each of the three risky assets that make up the tangency portfolio?

(b) (5 Points) What is the expected (not excess) return of the tangency portfolio? What is the standard deviation of the tangency portfolio?

(c) (5 Points) What is the expected (not excess) return of the hedge fund? What is the standard deviation of the hedge fund? (Hint: 100% leverage means a weight of 2 in the tangency portfolio.)

(d) (10 Points) What are the weights in the three risky assets alone (no leverage) that would allow you to match the expected return of the hedge fund? (Hint: You will need to find a point on the mean variance frontier.)

Extra Space:

Extra Space:

Extra Space:

Extra Space:

Extra Space:

Extra Space:

Useful Formulas

- Combining risk-free asset and a single risky asset in a portfolio:
  - Expected return:
  $$E[r_p] = wE[r_A] + (1-w)r_f$$
  - Standard deviation of the return:
  $$\sigma_p = w\sigma_A$$
  - Capital Allocation Line with one risky security:
  $$E[r_p] = r_f + \left(\frac{E[r_A] - r_f}{\sigma_A}\right)\sigma_P$$
  - Sharpe Ratio:
  $$\text{Sharpe Ratio of A} = \frac{E[r_A] - r_f}{\sigma_A}$$
- Portfolio with two risky assets B and C where w is the weight on B.
  - Excess return on B: $E[r_B^e] = E[r_B] - r_f$
  - Expected Return:
  $$E[r_P] = w * E[r_B] + (1-w) * E[r_C]$$
  - Variance
  $$\begin{aligned}\sigma_P^2 &= w^2\sigma_B^2 + (1-w)^2\sigma_C^2 + 2w(1-w)cov(r_B, r_C)\\ &= w^2\sigma_B^2 + (1-w)^2\sigma_C^2 + 2w(1-w)\rho_{B,C}\sigma_B\sigma_C\end{aligned}$$
- Portfolio statistics for many risky assets
  - Mean:
  $$E[r_p] = \mu_p = w'\mu = \sum_{i=1}^{N} w_i E[r_i]$$
  - Variance:
  $$\sigma_p^2 = w'\Sigma w = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j}$$
  - where μ is the vector of expected returns for each asset, w is a vector of portfolio weights, w_i represents the portfolio weight for security i, Σ is the variance covariance matrix of all the risky securities, $\sigma_{i,j}$ is the covariance between security i and j.

- Portfolio allocation for investors with quadratic utility and no outside income.
  - Investors have quadratic utility

$$u(\mu, \sigma) = \mu - \frac{A}{2}\sigma^2$$

    where μ is their portfolio expected return, σ is the standard deviation of their portfolio and A is their risk aversion.
  - The optimal portfolio weight in the risky asset when mixing risky asset i with the risk free asset is

$$w_i = \frac{E(r_i) - r_f}{A\sigma^2(r_i)}$$

    where $E(r_i)$ is expected return on the risky asset, $\sigma^2(r_i)$ is the variance of the risky asset and r_f is the return on the risk free asset.

- Total wealth portfolio statistics for investors with outside income

$$\begin{aligned}
\mu_T &= (f)E[r_p] + (1-f)\mu_j \quad \text{and} \\
\sigma_T^2 &= f^2\sigma_p^2 + (1-f)^2\sigma_j^2 + 2 * f * (1-f) * cov(r_p, r_j) \\
&= f^2\sigma_p^2 + (1-f)^2\sigma_j^2 + 2 * f * (1-f) * w * \rho * \sigma_j * \sigma_m
\end{aligned}$$

  where μ_T is the expected return on total wealth, σ_T is the standard deviation on total wealth, $E[r_p]$ is the expected return on the investors financial portfolio, σ_p is the standard deviation of the investors financial portfolio, μ_j is the expected return to outside income, σ_j is the standard deviation of outside income, ρ is the correlation between outside income and the market, σ_m is the standard deviation on the market, f is the fraction of total wealth in financial assets, and w is the weight in the risky assets (the market) in the investors financial portfolio.

- Bond Pricing
  - Yield to maturity, y, on a coupon bond with T periods to maturity, price P, coupons C and face value F:

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \cdots + \frac{C+F}{(1+y)^T}$$

  - Yield to maturity (YTM_j) on a zero coupon bond with maturity j periods, price P_j and face value F:

$$r_j = YTM_j = \left(\frac{F}{P_j}\right)^{\frac{1}{j}} - 1$$

  - Price of risk-free cash flows C_1, C_2, ... C_T (present value formula):

$$P = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \cdots + \frac{C_T}{(1+r_t)^T}$$

- Forward rate between period j and period $j+1$

$$f_{j,j+1} = \frac{(1+r_{j+1})^{j+1}}{(1+r_j)^j} - 1 = \frac{P_j}{P_{j+1}} - 1$$

- Liquidity Premium Hypothesis:

$$f_{j,j+1} = E[r_{1,j}] + \text{term premium}$$

- Macaulay Duration of risk-free cash flows C_1, C_2, ... C_T:

$$D = \left(\frac{1}{P}\right)\left(\frac{1 * C_1}{1+y} + \frac{2 * C_2}{(1+y)^2} + \cdots + \frac{T * C_T}{(1+y)^T}\right)$$

- Modified Duration:

$$\begin{aligned}\text{Mod. D} &= \left(\frac{1}{1+y}\right)\left(\frac{1}{P}\right)\left(\frac{1 * C_1}{1+y} + \frac{2 * C_2}{(1+y)^2} + \cdots + \frac{T * C_T}{(1+y)^T}\right) \\ &= \left(\frac{1}{1+y}\right)(\text{Macaulay Duration})\end{aligned}$$

- Duration of a portfolio where w_A and w_B are portfolio weights and D_A and D_B are the duration of the components

$$D_P = w_A * D_A + w_B * D_B$$

- Duration matching

$$D_A * \$\text{Assets} = D_L * \$\text{Liabilities}$$

- Convexity

$$\text{Convexity} = \frac{1}{P(1+y)^2}\left(\frac{(1^2+1) * C_1}{1+y} + \frac{(2^2+2) * C_2}{(1+y)^2} + \cdots + \frac{(T^2+T) * C_T}{(1+y)^T}\right)$$

- Approximate price change in response to a yield change Δy using only duration:

$$\%\Delta\text{Price} = \frac{dP}{P} = -(\text{Modified Duration}) * \Delta y$$

- Approximate price change in response to a yield change Δy using duration and convexity:

$$\%\Delta\text{Price} = \frac{dP}{P} = -(\text{Modified Duration}) * \Delta y + \frac{\text{Convexity} * (\Delta y)^2}{2}$$

- Statistics

- Relation between correlation and covariance, etc.

$$cov(x, y) = \rho * \sigma_x * \sigma_y$$

- Portfolio Statistics for Many Risky Assets
  - Mean:
$$E[r_p] = \mu_p = w'\mu = \sum_{i=1}^{N} w_i E[r_i]$$
  - Variance:
$$\sigma_p^2 = w'\Sigma w = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j}$$
  - where μ is the vector of expected returns for each asset, w is a vector of portfolio weights, w_i represents the portfolio weight for security i, Σ is the variance covariance matrix of all the risky securities, $\sigma_{i,j}$ is the covariance between security i and j.

- Mean Variance Analysis:
  - Tangency Portfolio Weights:
$$w_T = \frac{\Sigma^{-1}\mu^e}{\vec{1}'\Sigma^{-1}\mu^e}$$
  - Minimum Variance Portfolio Weights:
$$w_{mv} = \frac{\Sigma^{-1}\vec{1}}{\vec{1}'\Sigma^{-1}\vec{1}}$$
  - where μ^e is the expected excess return on each risky asset, Σ is the variance-covariance matrix of all the risky assets and Σ^{-1} is the inverse of the variance-covariance matrix, $\vec{1}$ is a vector of ones of the same length as the number risky assets.

- Capital Asset Pricing Model (CAPM):
  - Excess Returns: $r_i^e = r_i - r_f$
  - Beta:
$$\beta_j = \frac{cov(r_j^e, r_{mkt}^e)}{\sigma^2(r_{mkt}^e)}$$
  - Beta of a portfolio with weights w_i on stock i and w_j on stock j: $\beta_P = w_i\beta_i + w_j\beta_j$
  - Realized excess returns: $r_{j,t}^e = \beta_j(r_{mkt,t}^e) + \epsilon_{j,t}$
  - Expected excess return: $E(r_j^e) = \beta_j[E(r_{mkt}^e)]$ (This gives the Security Market Line.)

- Regression equation: $r^e_{j,t} = \alpha_j + \beta_j(r^e_{mkt,t}) + \epsilon_{j,t}$
- Variance of return:

$$\sigma_j^2 = \beta_j^2\sigma_{mkt}^2 + \sigma_\epsilon^2 = \text{systematic risk} + \text{non-systematic risk}$$

- Alpha (α is zero if the CAPM is true!)

$$E(r^e_j) = \alpha + \beta_j[E(r^e_{mkt})]$$

- Alpha of a portfolio with weights w_i on stock i and w_j on stock j: $\alpha_P = w_i\alpha_i + w_j\alpha_j$
- Relevant covariance implied by CAPM if all non-diversifiable comovement comes only from covariance with market:

$$cov(r^e_A, r^e_B) = \beta_A\,\beta_B\,\sigma^2(r^e_{mkt})$$

- Fama French 3 Factor Model
  - Realized excess returns

$$r^e_{j,t} = \beta_{j,mkt}r^e_{mkt,t} + \beta_{j,smb}r_{smb,t} + \beta_{j,hml}r_{hml,t} + \epsilon_{j,t}$$

  - Expected excess returns

$$E[r^e_j] = \beta_{j,mkt}E[r^e_{mkt}] + \beta_{j,smb}E[r_{smb}] + \beta_{j,hml}E[r_{hml}]$$

  - Regression equation

$$r^e_{j,t} = \alpha_j + \beta_{j,mkt}r^e_{mkt,t} + \beta_{j,smb}r_{smb,t} + \beta_{j,hml}r_{hml,t} + \epsilon_{j,t}$$

  - Alphas and betas of portfolios add up as weighted sums just like under the CAPM.
- Regressions
  - t-statistics

$$\text{tstat} = \frac{\text{measured} - \text{null}}{\text{standard error}}$$