Q.1

a) $\frac{x-3}{(x^2-4)} + \frac{x-2}{x^2-x-6}$ $\qquad x^2-x-6 = (x-3)(x+2)$

$\frac{x-3}{(x+2)(x-2)} + \frac{x-2}{(x-3)(x+2)}$

$\frac{(x-3)(x-3)}{(x+2)(x-2)(x-3)} + \frac{(x-2)(x-2)}{(x-3)(x+2)(x-2)}$

$\frac{x^2-6x+9+x^2-4x+4}{(x+2)(x-2)(x-3)}$

$= \frac{2x^2-10x+13}{(x+2)(x-2)(x-3)}$

Restrictions

$x \neq 2 \qquad x \neq -2$

$x \neq 3$

b) $\frac{x-1}{x^2+2x-3} \div \frac{x+2}{x^2+5x+6}$

$\frac{\cancel{x-1}}{\cancel{(x-1)}(x+3)} \div \frac{\cancel{x+2}}{\cancel{(x+2)}(x+3)}$

$\frac{1}{\cancel{x+3}} \times \frac{\cancel{x+3}}{1} = \underline{\underline{1}}$

Restrictions

$x \neq 1 \qquad x \neq -2.$

$x \neq -3$

Q.2 $x = 2+\sqrt{5}$ $\quad x = 2-\sqrt{5}$ $\quad (7, -40)$

$$Q(x) = (x-(2+\sqrt{5}))(x-(2-\sqrt{5}))$$

$$-40 = a(7-2-\sqrt{5})(7-2+\sqrt{5})$$

$$-40 = a(5-\sqrt{5})(5+\sqrt{5})$$

$$-40 = a(25-5)$$

$$-40 = 20a$$

$$a = -2$$

$$Q(x) = -2(x-2-\sqrt{5})(x-2+\sqrt{5})$$

$$= -2(x^2 - 2x + x\sqrt{5} - 2x + 4 - 2\sqrt{5} - x\sqrt{5} + 2\sqrt{5} - 5)$$

$$= -2(x^2 - 4x - 1)$$

$$Q(x) = -2x^2 + 8x + 2.$$

Q.3

$$y = -3\left(\frac{1}{2}\right)^{x-1} + 2$$

A vertical shift upwards by 2 units.

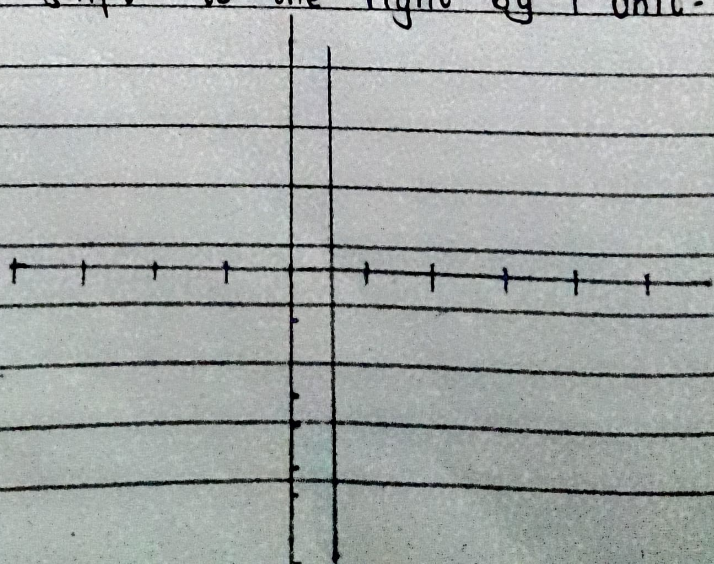
A vertical stretch by 3 ~~units~~.

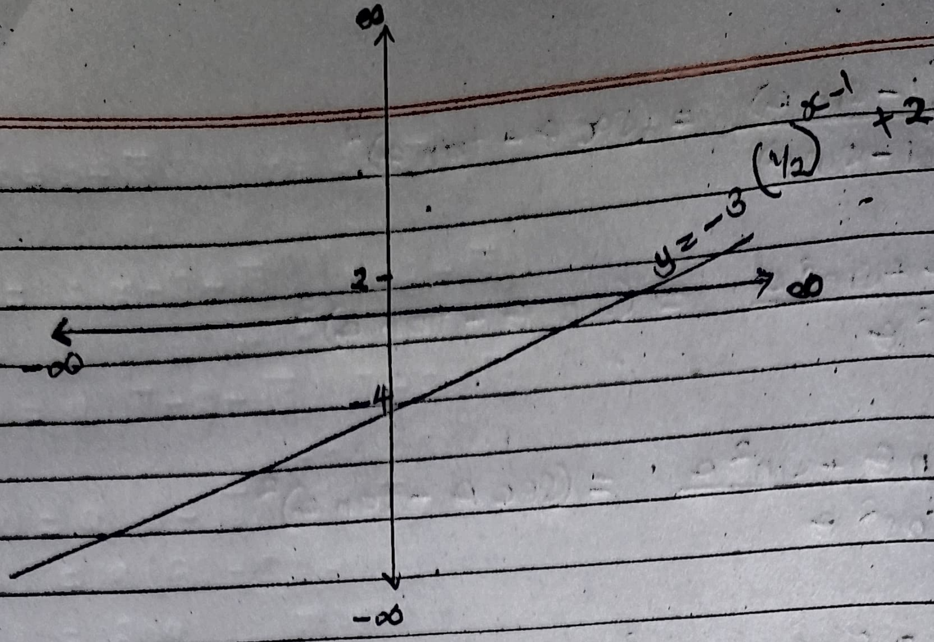
A horizontal shrink by $\frac{1}{2}$.

A horizontal shift to the right by 1 unit.

Domain

Range





Domain $[-\infty, \infty)$

Range $(-\infty, 2]$

Q.4 $p = 100(0.975)^{d}$

a) $p = 100(0.975)^{100}$

$= 7.9517\%$

b) $p = 100(0.975)^{40}$

$= 36.3232\%$

$p = 100(0.975)^{60}$

$= 21.8915\%$

$36.3232 - 21.8915$

$= 14.4317\%$ change

Q.5 Prove $\frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}} = \sec\theta - \tan\theta$.

Squaring both sides;

$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

Taking the LHS;

$$\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = (\sec\theta - \tan\theta)^2$$

$$\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta} = (\sec\theta - \tan\theta)^2$$

$$\frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta} = (\sec\theta - \tan\theta)^2$$

$$\frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = (\sec\theta - \tan\theta)^2$$

$$\sec^2\theta - \frac{2\sin\theta}{\cancel{\tan\theta}\ \cos\theta} \cdot \frac{1}{\cos\theta} + \tan^2\theta = (\sec\theta - \tan\theta)^2$$

$$\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta = (\sec\theta - \tan\theta)^2$$

$$(\sec\theta - \tan\theta)^2 = (\sec\theta - \tan\theta)^2$$

~~Hence~~ Taking the square root on both sides;

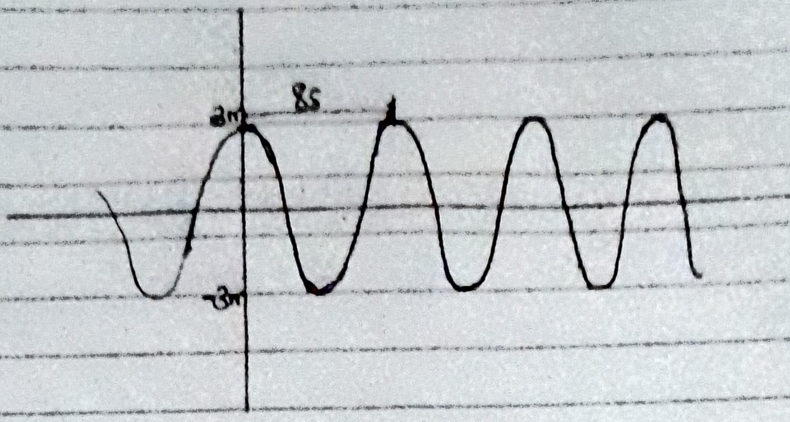
$$(\sec\theta - \tan\theta) = (\sec\theta - \tan\theta)$$

Hence proved.

#

Q.6.

a)



b) $f(x) = A \cos\left(\frac{2\pi}{\lambda} x\right)$

$A = 3m$

$\lambda = 8s$

$f(x) = 3 \cos \frac{2\pi}{8} x$

$f(x) = 3 \cos \frac{\pi}{4} x$

c) At 17s

$f(x) = 3 \cos \frac{\pi}{4} \times 17$

$= 2.12m$ above sea level.