CHAPTER 9

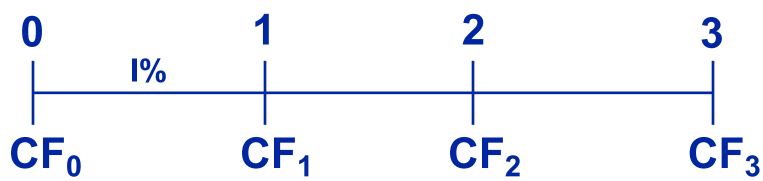
Time Value Analysis

The financial (monetary) value of any investment is the current value of its expected future cash flows. Such valuations typically involve cash flows that occur over time. One of the cornerstones of financial analysis is that the timing of cash flows, as well as their size, must be recognized.

Time Value of Money

- Time value analysis is necessary because money has time value.
  - A dollar in hand today is *worth more* than a dollar to be received in the future. Why?
  - Because of time value, the values of future dollars must be *adjusted* before they can be compared to current dollars.
- Time value analysis, or discounted cash flow analysis, constitutes the techniques used to account for the time value of money.

Time Lines



Tick marks designate ***ends*** of periods. Time 0 is the starting point (the beginning of Period 1); Time 1 is the end of Period 1 (the beginning of Period 2); and so on.

What is the FV after three years of a $100 lump sum invested at 10%?

- Finding future values (moving to the *right* along the time line) is called **compounding**.
- For now, assume interest is paid *annually*.

After 1 year:

$$FV_1 = PV + INT_1 = PV + (PV \times I)$$
$$= PV \times (1 + I)$$
$$= \$100 \times 1.10 = \$110.00.$$

After 2 years:

$$FV_2 = FV_1 + INT_2$$
$$= FV_1 + (FV_1 \times I) = FV_1 \times (1 + I)$$
$$= PV \times (1 + I) \times (1 + I) = PV \times (1 + I)^2$$
$$= \$100 \times (1.10)^2 = \$121.00.$$

After three years:

$$FV_3 = FV_2 + I_3$$
$$= PV \times (1 + I)^3$$
$$= 100 \times (1.10)^3$$
$$= \$133.10.$$

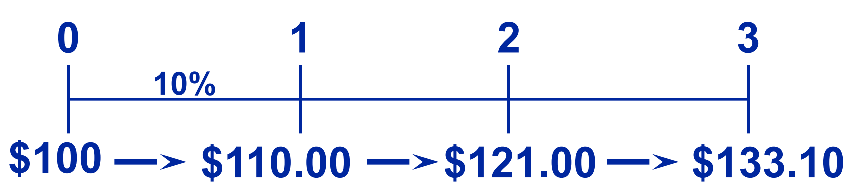
In general,

$$FV_N = PV \times (1 + I)^N.$$

Three Primary Methods to Find FVs

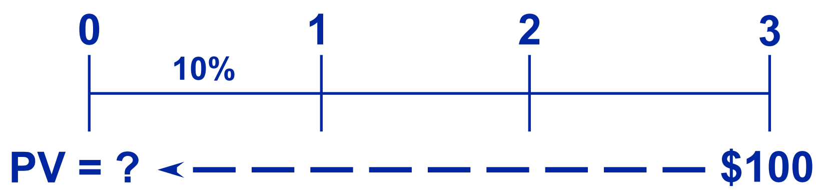
- Solve the FV equation using a *regular (nonfinancial) calculator*.
- Use a *financial calculator*; that is, one with financial functions.
- Use a *computer* with a *spreadsheet program* such as Excel or Quattro Pro.

Nonfinancial Calculator Solution



$100 × 1.10 × 1.10 × 1.10 = $133.10.

What is the PV of $100 due in three years if I = 10%?



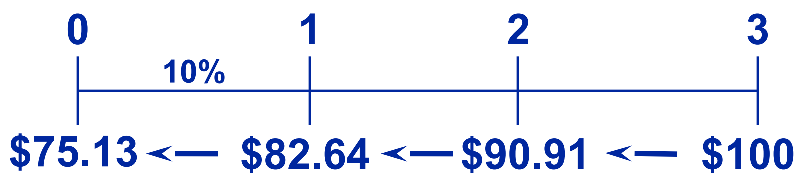
Finding present values (moving to the *left* along the time line) is called discounting.

Solve $FV_N = PV \times (1 + I)^N$ for PV

$PV = FV_N / (1 + I)^N$.

$PV = \$100 / (1.10)^3$
$= \$100(0.7513) = \$75.13$.

Time Line Solution



$100 ÷ 1.10 ÷ 1.10 ÷ 1.10 = $75.13.

Note that the calculated present value ($75.13), when invested at 10 percent for three years, will produce the starting future value ($100).

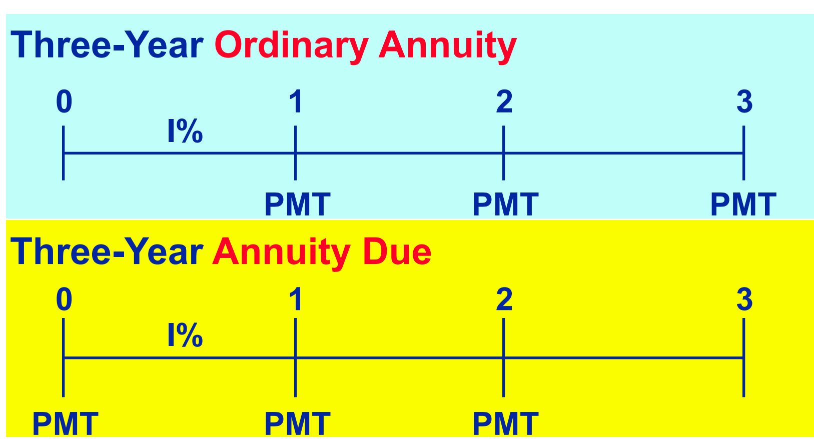
Opportunity Cost Rate

- **On the last illustration we needed to apply a discount rate. Where did it come from?**
  - **The discount rate is the opportunity cost rate.**
  - **It is the rate that could be earned on alternative investments of *similar risk*.**
  - **It does *not* depend on the source of the investment funds.**
- **We will apply this concept over and over in this course.**

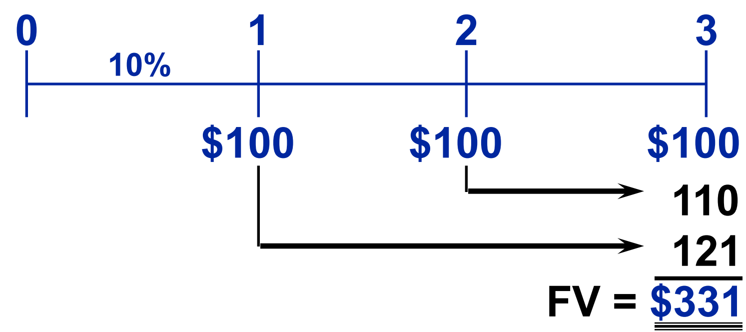
Opportunity Cost Rate (Cont.)

- The opportunity cost rate is found (at least in theory) as follows:
  - Assess the riskiness of the cash flow(s) to be discounted.
  - Identify ***security investments*** that have the same risk. **Why securities?**
  - Estimate the return expected on these similar-risk investments.
- When applied, the resulting PV provides a return ***equal*** to the opportunity cost rate.
- In most time value situations, benchmark opportunity cost rates are known.

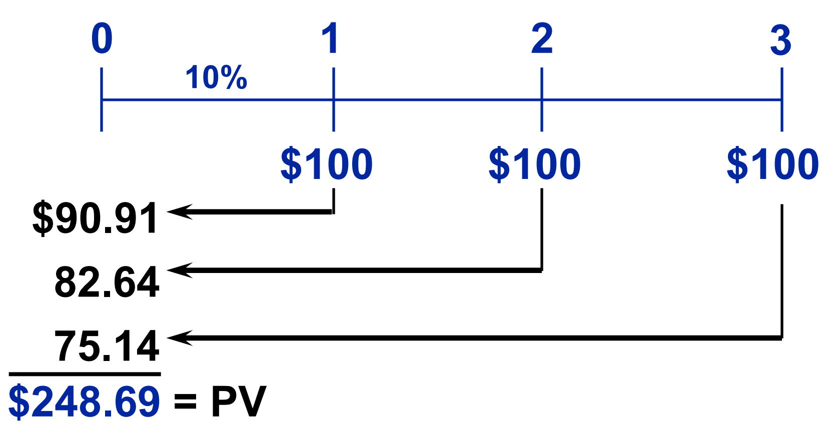
Types of Annuities



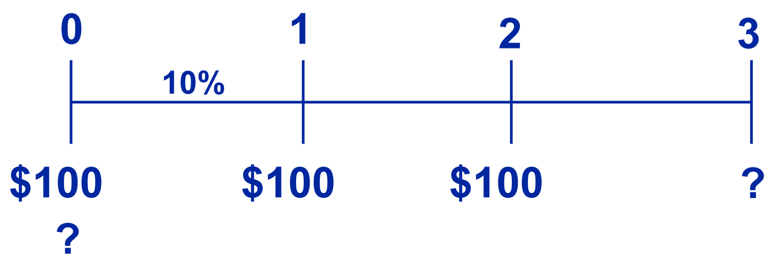
What is the FV of a three-year ordinary annuity of $100 invested at 10%?



What is the PV of the annuity?



What are the FV and PV if the annuity were an annuity due?



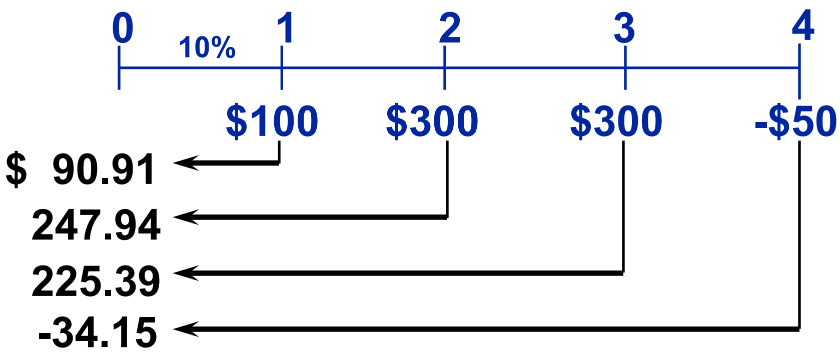
Perpetuities

- A perpetuity is an annuity that lasts forever.
- What is the present value of a perpetuity?

$$PV\ (Perpetuity) = \frac{PMT}{I}.$$

- What is the future value of a perpetuity?

Uneven Cash Flow Streams



$530.09 = PV (often called net PV [NPV])

Investment Returns

- The financial performance of an investment is measured by its return.
  - Time value analysis is used to calculate investment returns.
  - Returns can be measured either in dollar terms or in rate of return terms.
- Assume that a hospital is evaluating a new MRI. The project's expected cash flows are given on the next slide.

MRI Investment Expected Cash Flows (in thousands of dollars)

0	1	2	3	4
-$1,500	$310	$400	$500	$750

? Where do these numbers come from?

Simple Dollar Return

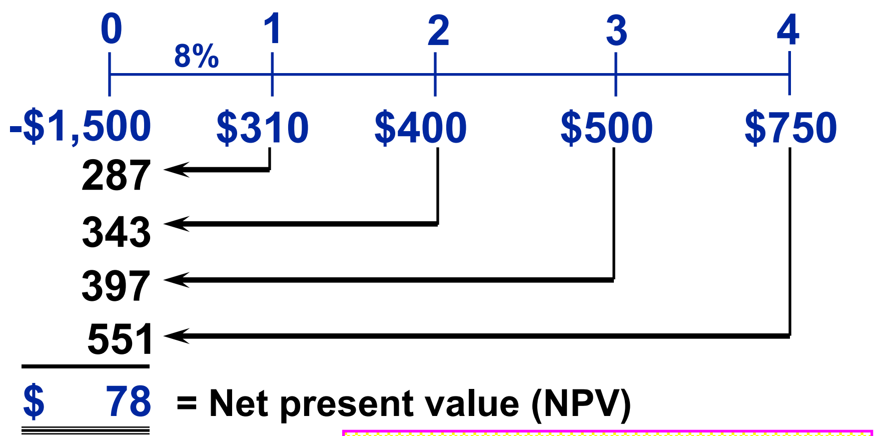
0	1	2	3	4
-$1,500	$310	$400	$500	$750

310
400
500
750

$ 460 = Simple dollar return

? Is this a good measure?

Discounted Cash Flow (DCF) Dollar Return



? Where did the 8% come from?

DCF Dollar Return (Cont.)

- The key to the effectiveness of this measure is that the discounting process automatically recognizes the ***opportunity cost of capital***.
- An NPV of zero means the project just earns its opportunity cost rate.
- A positive NPV indicates that the project has positive financial value after opportunity costs are considered.

Rate of (Percentage) Return

0	1	2	3	4
10%				
-$1,500	$310	$400	$500	$750

282 ← $310

331 ← $400

376 ← $500

511 ← $750

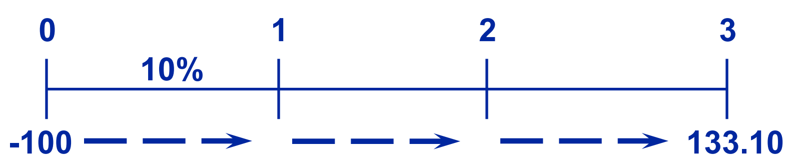
$ 0.00 = NPV, so rate of return = 10.0%.

Rate of Return (Cont.)

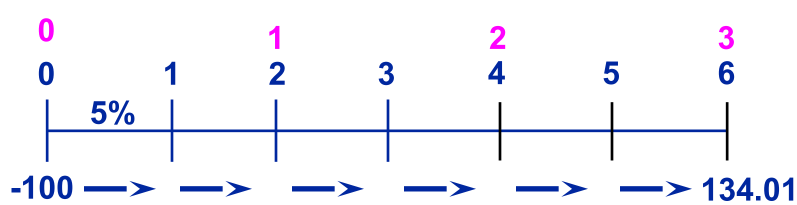
- In capital investment analyses, the rate of return often is called internal rate of return (IRR).
- In essence, it is the percentage return expected on the investment.
- To interpret the rate of return, it must be *compared* to the opportunity cost of capital, in this case 10% versus 8%.

Intra-year Compounding

- Thus far, all examples have assumed annual compounding.
- When compounding occurs intra-year, the following occurs:
  - Interest is earned on interest during the year (more frequently).
  - The ***future value*** of an investment is ***larger*** than under annual compounding.
  - The ***present value*** of an investment is ***smaller*** than under annual compounding.



Annual: $FV_3 = 100 \times (1.10)^3 = 133.10$.



Semiannual: $FV_6 = 100 \times (1.05)^6 = 134.01$.

Effective Annual Rate (EAR)

- **EAR is the *annual rate*, which causes the PV to grow to the same FV as under intra-year compounding.**
- **What is the EAR for 10%, semiannual compounding?**
  - **Consider the FV of $1 invested for one year. FV = $1 × $(1.05)^2$ = $1.1025.**
  - **EAR = 10.25%, because this rate would produce the same ending amount ($1.1025) under annual compounding.**

The EAR Formula

$$EAR = \left(1 + \frac{I_{Stated}}{M}\right)^M - 1.0$$

$$= \left(1 + \frac{0.10}{2}\right)^2 - 1.0$$

$$= (1.05)^2 - 1.0 = 0.1025 = 10.25\%.$$

EAR of 10% at Various Compounding

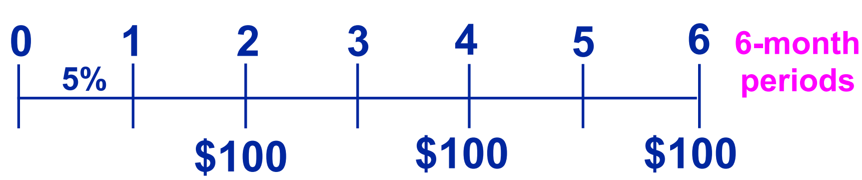
EAR_{Annual} = **10%.**

$EAR_Q = (1 + 0.10/4)^4 - 1.0$ = **10.38%.**

$EAR_M = (1 + 0.10/12)^{12} - 1.0$ = **10.47%.**

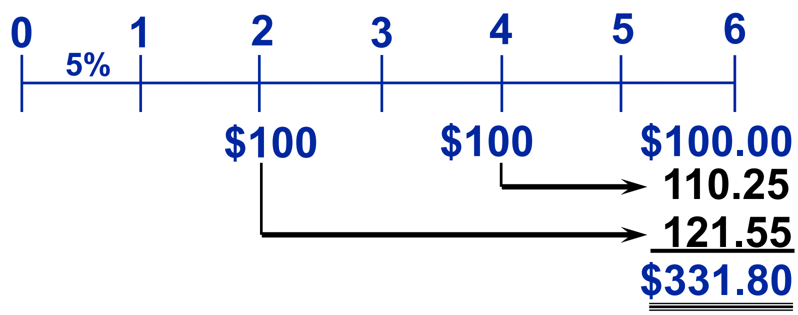
$EAR_{D(360)} = (1 + 0.10/360)^{360} - 1.0$ = **10.52%.**

Using the EAR



Here, payments occur *annually*, but compounding occurs *semiannually*, so we can not use normal annuity valuation techniques.

First Method: Compound Each CF



That's all for Chapter 9