

4. *Turbine-electric (alternating current).* Where low-pressure process steam, not readily available from other sources, is needed and where the simple co-ordination control of the a-c system is sufficient.

In addition to these considerations, the

type of co-ordination control desired may be a decisive factor in selecting the type of drive. Table I lists the various kinds of co-ordination control with indication of suitability of the control for the different

kinds of drives, based on accuracy adaptability, and stability.

No Discussion

Fundamental Basis for Distance Relaying on 3-Phase Systems

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FOR THE PROTECTION of transmission lines against short circuits and ground faults, it is desirable to have a system of relays which will divide all faults on the line it protects, regardless of type or system conditions, into two classes: one, those which occur between the relaying station and the next section-alizing point on that line, and two, all other faults. It is also desirable that this classification be performed instantaneously, so that the section in trouble may be isolated immediately, thus reducing the damage at the point of fault and the shock to the remainder of the system, and in general permitting operation nearer the stability limit. The quantities which may be utilized for classification are currents, voltages, the phase relations between them, and time. The demand for high speed of fault isolation, rendered acute by the development of high-speed circuit breakers, largely has disqualified time as a fundamental means of discrimination, leaving only the other three.

For protection of single lines, the nearest approach to this ideal, exclusive of pilot-wire or superposed high-frequency methods, has been obtained by the use of distance relays. For parallel lines a somewhat closer approach has been obtained by means of balanced-current relays.

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However, additional protection is required for single-line operation and to take care of the possibility of simultaneous faults on both lines, for which distance relays are well suited. Therefore, if the distance relays are made sufficiently accurate and rapid, the balanced-current relays may be dispensed with or provided for backup protection only. A systematic and comprehensive analysis of the voltages and currents which may be used to operate the distance relays, to determine their suitability and the results which may be expected, has not appeared previously. The aim of this paper is to provide such an analysis.

Distance relays consist of two basic types: impedance relays and reactance relays. The impedance relay compares the magnitudes of the voltage and the current, the ratio being the indicated impedance. The reactance relay compares the magnitude of that component of the voltage 90 degrees out of phase with the current and the magnitude of the current, the ratio being the indicated reactance. Since the original presentation of this paper, several modified types of relays have been developed, having characteristics which are modifications of, or combinations of, the characteristics of impedance or reactance relays. The purpose of these special types has been to minimize the danger of undesired relay operation during load swings or other system disturbances, while retaining or improving the ability of the relay to operate for faults in the protected zone with fault resistance present.

Under line-to-line fault conditions on an individual single-phase line, the indicated impedance is equal to the actual impedance of the circuit from the relay to the fault, plus the effect of any fault im-

pedance which is present. On polyphase systems the ratio is not necessarily equal to the circuit impedance, but may be affected in various ways by the type of fault, the system load, and other conditions external to the section being protected, as well as by the fault impedance. As the impedance in the fault is primarily resistance, it generally is assumed that a relay operating on the reactive component of the impedance will be unaffected by the fault resistance. On a radial single-phase system this is quite generally true, but, if the fault is fed from two or more points or if a polyphase system is involved, it does not follow necessarily. Current flowing through the fault resistance produces a voltage drop in phase with it. This voltage drop is added to the line drop and thus affects the voltage at the relay. Unless the current through the relay is exactly in phase with the current through the fault, the fault resistance drop will have a component 90 degrees out of phase with the relay current thus producing an effect similar to reactance in the line. This apparent reactance can be either positive or negative thus making possible an increase or decrease in the indicated reactance and adversely affecting the relay operation. If the fault resistance becomes very large in comparison with the reactance of the transmission line section, this effect becomes quite serious and may prevent the application of the relay in some cases, particularly on very short lines.

Because the circuit impedances are not absolutely invariable and the relaying indications not an absolutely accurate measure of the distance to the fault, it has been found impractical with distance relays to classify all faults into the two classes specified. For faults near the remote end of the section, the relay is unable to determine whether the fault is just within or just beyond the end of the section. Hence, an intermediate class has been introduced to include faults in the immediate neighborhood of the end of the section on either side. If a fault falls in the first class, the circuit breaker is tripped instantly. If a fault falls in the second or intermediate class, a definite

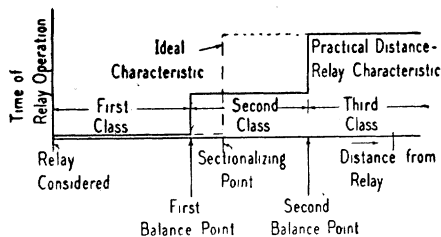


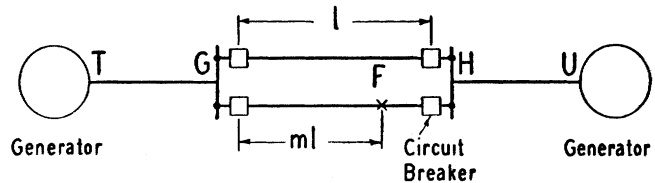
Figure 1. Ideal and practical distance relay characteristics

time delay is introduced before the circuit breaker is tripped, and if the fault falls in the third class, the circuit breaker either is not tripped or is tripped only after a considerably greater time delay. This is shown in Figure 1. If the fault is beyond the end of the section, it will be cleared by other circuit breakers and relays before the expiration of the time delay, and the relay will reset. If the fault is just within the section, the relay will trip the circuit breaker at the end of the time interval. Thus, the final classification for faults in the intermediate group is made in conjunction with the operation of other circuit breakers and relays during a short time interval.

When impedance or reactance relays are used, the fault belongs in the first class, if the indicated impedance or reactance is below a definite lower limit; in the intermediate class, if the indicated quantity lies within two given limits; and in the third class, if it exceeds the upper limit. It will be convenient to call the points at which faults must occur in order to produce the limiting values of indicated impedance or reactance "balance points," as for such faults the relay just balances between one type of operation and another. To achieve the ideal, it is necessary to make the first balance point coincide exactly with the remote end of the section. In practice this is not possible, but in order to prevent false operation, and give full protection, the first balance point never must extend into the adjoining section, and the second balance point in the adjoining section never must extend into any other section. The smaller the variations in balance point position, the closer the balance points may be located to the remote end of the section, and the more nearly the relaying system approaches the ideal.

It is not essential for the second balance point to be located close to the end of the first section, as advantage may be taken of its presence in the adjoining section to provide backup protection for a portion of the second section, in case the circuit breakers in that section fail, for

Figure 2. Single-line diagram—assumed equivalent system



any reason, to open when they should. However, it is highly desirable that the variation be kept to a minimum, so that a short section of line may follow a long one without danger of the balance point moving out of the section, or extending beyond the first balance point of the relay protecting that section.

If the quantity indicated by the relay is constant for all faults in any given location, but increases as the distance to the fault increases, and decreases as the distance to the fault decreases, it is obvious that the balance points may be fixed accurately. If the quantity is variable for different faults at the same location, the balance points will shift an amount depending upon the maximum variation. It is, therefore, important to eliminate or reduce the variations in indicated impedance or reactance as far as is practicable.

The quantities which would be indicated by the two basic types of relays under different fault conditions are determined in the paper in terms of the circuit constants and fault currents, and a comparison is made to show the possible variations. The methods which will result in smaller variations are pointed out.

For the purpose of developing the necessary formulas, a typical 2-circuit transmission system has been assumed, as shown in Figure 2, with a source of power at each end of the line. The 2-circuit line between stations G and H has been shown in detail, with bussing of the two lines assumed at these points. The remainder of the system at each end has been represented by a single equivalent circuit and a single generator. In general, any section of a system may be represented in this manner for short circuits outside the section so simplified, although the constants of the equivalent circuit and equivalent generator may change. If there are ties between the two generating stations other than the lines shown, provided these ties are so remote that they do not have appreciable mutual coupling with the two lines considered, the effects may be taken into account by giving to the constants of the equivalent circuit and the equivalent generator variable values, such that at every instant the voltages and currents flowing from the simplified portion of the system into stations G and

H are the same as those occurring under the actual conditions. This is always possible, although the practical application may be quite difficult. For our purposes, it is necessary only to know that the representation is possible, the actual values of the equivalent circuit constants being required only in special cases. Adequate neutral grounding is assumed at both ends of the system, although from the standpoint of the following theory it may be either solid or through impedance.

Shunt loads and distributed capacitance have been neglected, for simplicity, in line with the usual practice in the calculation of short-circuit currents, although later the effects of loads will be discussed. However, the simplification of assuming both generating sources in phase is not permissible, as phase relations have an important bearing on the results.

The use of symmetrical components for the determination of short-circuit currents during unbalanced faults is now well known, but the application to the determination of voltage conditions is not so widespread, due to the fact that the voltages are not required so often. For present purposes the voltages at relaying points are of particular importance, and, as the relaying voltages generally will be compared with the relaying currents, it is desirable to know the voltages in terms of the fault currents and the constants of the circuit, instead of in absolute values, thus facilitating the comparison. In the appendixes, the voltages at relaying points, line to neutral and line to line, as well as their symmetrical components, have been worked out in terms of the fault currents and circuit and fault impedances for all the common types of faults. In the body of the paper these voltages will be compared with the fault currents to determine the impedance or reactance indicated by the relay which should be operated for the particular fault, and comparisons drawn for possible relay connections.

The notation used is that of symmetrical components with additional subscripts to indicate the particular point or path involved. The notation is given in Appendix I.

For simplicity in writing the equations, the single-subscript method of indicating delta voltages has been used. The rela-

tion between the single-subscript notation, using capital-letter subscripts, and the more common double-subscript notation, using lower-case subscripts, is indicated in Figure 3. In all cases voltages represent voltage drops. When a single lower-case subscript, a , b , or c , is used, the voltage drop from that phase to neutral is intended. When two lower-case subscripts are used, the voltage drop from the first subscript to the second is meant. The corresponding relation for the single capital subscript is shown in Figure 3. A small plus sign indicates the terminal of higher potential when the corresponding voltage is positive.

It is necessary to set up the sequence networks for the assumed system. The networks are shown in Figures 4, 5, and 6, and the steps involved are explained in Appendix II. Here also the fault currents at relaying stations are set up in terms of their symmetrical components for the various types of faults. Certain instrument current transformer connections, which will be used, give different relaying currents, and these likewise are worked out assuming a 1-to-1 transformer ratio. The actual relaying currents may be obtained by dividing by the transformer ratio. The connections considered are shown in Figures 7, 8, and 9.

The use of a single set of relays for protection against all types of faults has not been found possible yet. One set of relays is required to protect against faults involving two or more line conductors, and another set of relays is needed for protection against faults between one conductor and ground. It is also possible, by means of a set of transfer relays, to use a single set of distance relays, connected in one way for faults involving two or more conductors, and in another way for faults involving one conductor and

ground. In the following discussion, the faults have been divided into two general types, the first consisting of line faults involving two or more conductors, and the second consisting of ground faults involving one conductor and ground.

Line Faults— Star-Current Relaying

The earlier connection of impedance or reactance relays for protection against line faults made use of the delta voltage and the line current for operating the relay. The phase relations usually were chosen so that the current lagged the voltage by 30 degrees at unity power factor. This connection is shown schematically in Figure 10, only the connections for the impedance element being given. By using the quantities given in the appendixes, the impedance or reactance given by this connection may be determined. In setting up the sequence expressions, line faults involving only two conductors are assumed to occur between conductors b and c . The delta voltage of this phase at the relaying point G is then E_{AG} . The line current which lags the voltage by 30 degrees, at unity power factor and flows from G to the fault is $(-I_{bGF})$.

LINE-TO-LINE FAULTS

Equation 106 of Appendix III gives the value of E_{AG} in terms of $(-I_{bGF})$ for this type of fault. Dividing this voltage by the relaying current $(-I_{bGF})$ gives the relay impedance Z_R , indicated by the relay at G which should trip, as

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_s}{2} \right) + 2 \frac{I_{bHF} R_s}{I_{bGF}} + \frac{I_{aGF} mZ_{1GH}}{I_{bGF}} \quad (1)$$

An impedance relay, of course, responds to the magnitude, and a reactance relay to the imaginary or reactive component of the foregoing vector expression.

To facilitate comparison it is also desirable to write this expression when the fault impedance is zero. In that case

$$Z_R = 2mZ_{1GH} + \frac{I_{aGF} mZ_{1GH}}{I_{bGF}} \quad (2)$$

If the circuit breaker at H is open, I_{bHF} and I_{aGF} are zero, and equation 1 becomes

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_s}{2} \right) \quad (3)$$

With no fault resistance this becomes

$$Z_R = 2mZ_{1GH} \quad (4)$$

DOUBLE LINE-TO-GROUND FAULTS

Equation 130 of Appendix IV gives the value of E_{AG} in terms of $(-I_{bGF})$ for this type of fault. Dividing by $(-I_{bGF})$ gives the indicated impedance as

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_s}{2} \right) + 2 \frac{I_{bHF} R_s}{I_{bGF}} + \frac{I_{aGF} mZ_{1GH}}{I_{bGF}} - \frac{3I_{0GF} R_s}{I_{bGF}} \quad (5)$$

As in the preceding case, an impedance relay indicates the magnitude, and a reactance relay the imaginary component of the foregoing vector expression.

If the fault resistance is zero this becomes

$$Z_R = 2mZ_{1GH} + \frac{I_{aGF} mZ_{1GH}}{I_{bGF}} - \frac{3I_{0GF} mZ_{1GH}}{I_{bGF}} \quad (6)$$

If the circuit breaker at H is open, these expressions simplify, as I_{aGF} , I_{bHF} and I_{0HF} are zero, giving

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_s}{2} \right) - \frac{3I_{0GF} \left(mZ_{1GH} + \frac{R_s}{2} \right)}{I_{bGF}} \quad (7)$$

and for zero fault resistance giving

$$Z_R = 2mZ_{1GH} - \frac{3I_{0GF} mZ_{1GH}}{I_{bGF}} \quad (8)$$

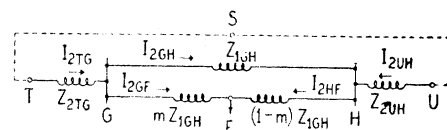


Figure 5 (above). Negative-sequence network for assumed system

Figure 6 (below). Zero-sequence network for assumed system

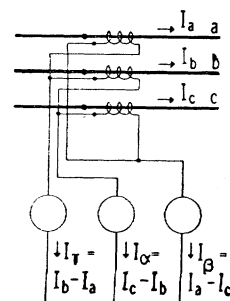
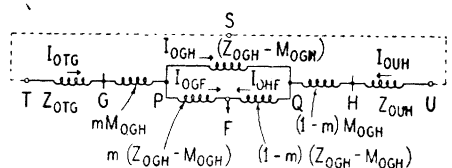


Figure 7. Delta-connected current transformers

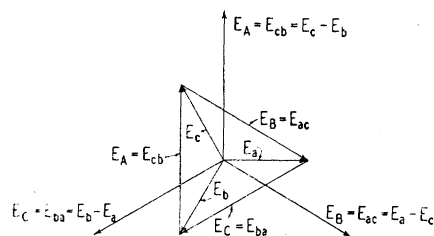
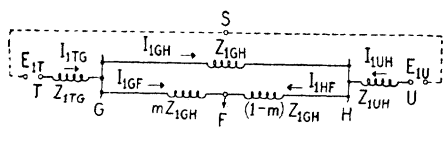


Figure 3 (above). Voltage vector diagram showing convention in choosing delta voltages

Figure 4 (below). Positive-sequence network for assumed system



THREE-PHASE FAULTS

Equation 146 of Appendix V gives the value of E_{AG} in terms of $(-I_{bGF})$ for this type of fault. Dividing by the relaying current $(-I_{bGF})$ gives

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_S}{2} \right) + 2 \frac{I_{bHF} R_S}{I_{bGF}} + \frac{I_{aGF}}{I_{bGF}} \left(mZ_{1GH} + \frac{R_S}{2} \right) + \frac{I_{aHF} R_S}{I_{bGF}} \quad (9)$$

If the fault resistance is zero, this becomes.

$$Z_R = 2mZ_{1GH} + \frac{I_{aGF}}{I_{bGF}} mZ_{1GH} \quad (10)$$

In this case, however, because of the fact that the fault is symmetrical, we have the following relations between the currents

$$I_{aGF}/I_{bGF} = I_{aHF}/I_{bHF} = \alpha$$

Also

$$2 + \alpha = \sqrt{3} e^{j30^\circ}$$

These expressions then become

$$Z_R = \sqrt{3} \left(mZ_{1GH} + \frac{R_S}{2} \right) e^{j30^\circ} + \sqrt{3} \frac{I_{bHF} R_S}{I_{bGF}} \quad (11)$$

and, when the fault resistance is zero,

$$Z_R = \sqrt{3} mZ_{1GH} e^{j30^\circ} \quad (12)$$

If the circuit breaker at H is open, I_{bHF} and I_{aHF} are zero, and equations 9 and 11 become, respectively,

$$Z_R = 2 \left(mZ_{1GH} + \frac{R_S}{2} \right) + \frac{I_{aGF}}{I_{bGF}} \left(mZ_{1GH} + \frac{R_S}{2} \right) \quad (13)$$

and

$$Z_R = \sqrt{3} \left(mZ_{1GH} + \frac{R_S}{2} \right) e^{j30^\circ}$$

Equations 10 and 12, being independent of currents from H , are unaffected by the opening of the circuit breaker at H .

COMPARISON

First consider that the fault impedance is zero. Comparison of equations 2, 6, and 10 shows that the first term of each, $2mZ_{1GH}$, is the same and, therefore, independent of the type of line fault. Moreover, it is equal to twice the line-to-neutral impedance between the relay and the fault, and so is proportional to the distance from the relay to the fault. However, each of these expressions contains one or more additional terms which involve not only the constants of the line but the currents in the remaining line conductor and in the ground, if any is flowing there, as well as

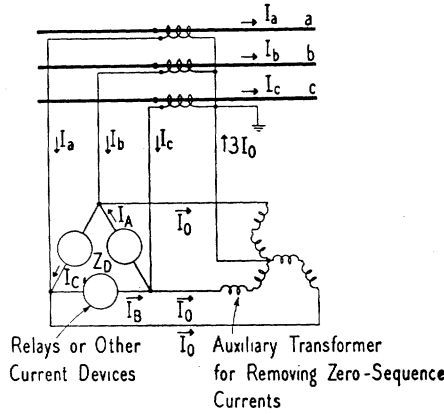
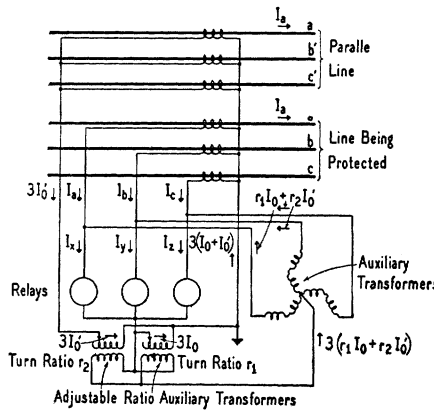


Figure 8 (above). Delta-connected device with zero-sequence filter

Figure 9 (below). Special current transformer connection for ground fault protection



the current in one of the faulted conductors. In a balanced 3-phase fault, definite relations exist between the currents in the line conductors, and the ground current is zero, so that in this case the value of the additional terms may be determined. This is done in equation 12. For both line-to-line and double line-to-ground faults, the currents in the unfaulted conductor and in the ground are determined by conditions external to the section being protected, and are subject to variations over wide limits. The current in the unfaulted conductor may be positive, negative, or zero, depending upon load conditions, exchange of synchronizing power, positions of circuit breakers at other points on the system, and so forth. The current in the ground during double line faults is also subject to considerable variation, depending upon the relative impedances to phase and to ground current. These variations cause corresponding variations in the value of Z_R , affecting both impedance and reactance relays. The exact range of variation is difficult to determine for the general case because of the wide range possible in the magnitudes

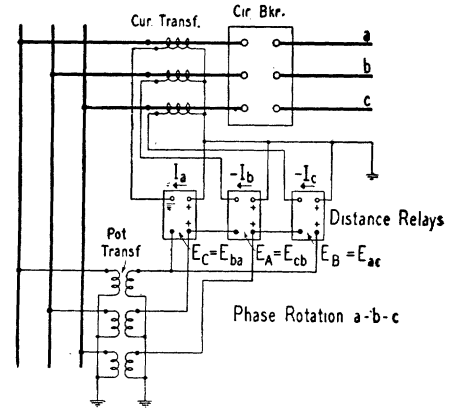
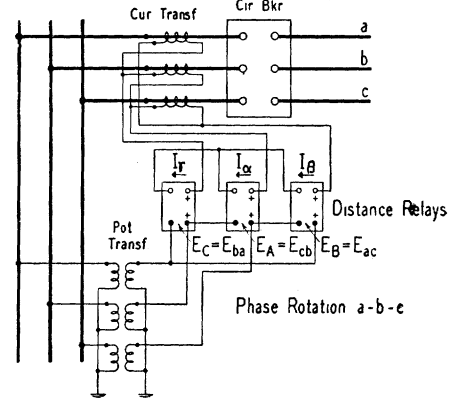


Figure 10 (above). Distance relay connections for line-fault protection, using line currents

Figure 11 (below). Preferred distance relay connections for line-fault protection using delta-connected current transformers



and phase angles of the currents involved. But a difference between maximum and minimum values of 25 per cent of the magnitude or reactive component of the first term is easily possible. As will be shown later, all the variations introduced by the preceding factors may be eliminated by changing the relay connections, so that it is not essential to analyze these limits very carefully.

After the circuit breaker at H has opened, Z_R becomes definite for both line-to-line and 3-phase faults, but is still variable for double line-to-ground faults. The corresponding values are given by equations 4, 8, and 12. As an impedance relay is unaffected by the 30-degree rotation of equation 12, the indicated impedance is reduced from $2mZ_{1GH}$ to $1.73 \times mZ_{1GH}$, a reduction of approximately 15 per cent. The reactance relay is affected by the phase relations, so that the reduction in reactance depends upon the phase angle of Z_{1GH} . If the phase angle is less than 75 degrees, the reduction is less than 15 per cent, and, if the angle is greater than 75 degrees, the reduction is greater than 15 per cent.

EFFECT OF FAULT RESISTANCE

The effect of resistance in the fault is to add two or more terms to the impedance expressions. One of these additional terms is always a pure resistance, and, therefore, adds at right angles to the reactance. The other term or terms are of the form $(I_n/I_{bGF})(R_S/2)$, where I_n may represent any one of several currents. If I_n and I_{bGF} are in phase, these terms also will be pure resistances and will add at right angles, but any phase angle difference will cause them to add vectorially at an angle differing from 90 degrees. Let the phase angles of I_n and I_{bGF} be θ_n and θ_{bGF} with respect to some arbitrary common reference. Then these terms may be expressed in the form

$$(I_n/I_{bGF})(R_S/2) \cos [(\theta_n - \theta_{bGF}) + j \sin (\theta_n - \theta_{bGF})]$$

It will be noted that the current symbols in this expression indicate the scalar values instead of the vector values. Due to the presence of the imaginary term in

the last parenthesis, this expression contains an apparent reactance equal to $(I_n/I_{bGF})(R_S/2) \sin (\theta_n - \theta_{bGF})$. If θ_n is greater than θ_{bGF} , this apparent reactance is positive, but, if θ_n is less than θ_{bGF} , indicating that I_n lags I_{bGF} , the apparent reactance is negative.

Unless the phase angle $(\theta_n - \theta_{bGF})$ is very large, the effect of these fault resistance terms is to increase the indicated impedance, making the fault appear more remote than it actually is, and therefore moving the balance point closer to the relay for that condition. With a reactance relay the first resistance term has no effect. However, the additional terms may cause either an increase or a decrease, depending upon phase relations between currents, so that the balance point may be shifted in either direction. As the balance point never must extend into the next section, allowance must be made for this condition in setting the relay, and, therefore, in general the balance point for a fault occurring without fault resistance may be made closer to the next sectionalizing point with an impedance relay than with a reactance relay.

Line Faults— Delta Current Relaying

The clue to a means of obtaining a constant relaying indication for all line faults is given by comparing equations 100, 124, and 143 which give the delta voltages at G on the faulted phases for line-to-line, double line-to-ground, and 3-phase faults

expressed in terms of the corresponding currents obtained from delta-connected current transformers.

For line-to-line faults equation 100 gives

$$E_{AG} = I_{\alpha GF} \left(mZ_{IGH} + \frac{R_S}{2} \right) + I_{\alpha HF} \frac{R_S}{2} \quad (15)$$

For double line-to-ground faults equation 124 gives

$$E_{AG} = I_{\alpha GF} \left(mZ_{IGH} + \frac{R_S}{2} \right) + I_{\alpha HF} \frac{R_S}{2} \quad (16)$$

For 3-phase faults equation 143 gives

$$E_{AG} = I_{\alpha GF} \left(mZ_{IGH} + \frac{R_S}{2} \right) + I_{\alpha HF} \frac{R_S}{2} \quad (17)$$

These expressions are all the same. It is evident, therefore, that if $I_{\alpha GF}$ is used for the relaying current instead of $(-I_{bGF})$, the same fundamental expression will be obtained for all types of line faults at a given location. If the circuit breaker at H is open, the relay indicates for all three types of faults

$$Z_R = mZ_{IGH} + \frac{R_S}{2} \quad (18)$$

This expression consists of an impedance term which is directly proportional to the distance to the fault and a variable fault resistance term. Hence, with the remote circuit breaker open, a reactance relay will receive a constant indication for all line faults which is proportional to the distance to the fault. The impedance relay will receive an indication which may vary somewhat due to the presence of fault resistance, and the variation depends upon the relative magnitudes of the line resistance plus fault resistance and the line reactance. However, it should be noted that only one half of the fault resistance from conductor to conductor should be used to compare with the positive-sequence, or line-to-neutral, impedance of the line in determining the effect of fault resistance on the indicated impedance.

When the circuit breaker at H is closed, the vector impedance is affected, of course, by the current from the remote end, and Z_R becomes

$$Z_R = mZ_{IGH} + \frac{R_S}{2} + \frac{I_{\alpha HF} R_S}{I_{\alpha GF} 2} \quad (19)$$

Here again, if $I_{\alpha HF}/I_{\alpha GF}$ is not a pure number, which may be the case due to differences in phase, the reactance relay also is affected, so that it is not altogether free from the effects of fault resistance so long as the fault is fed from two or more points. However, all other fundamental-frequency variations are eliminated, rendering the relay free from conditions of the

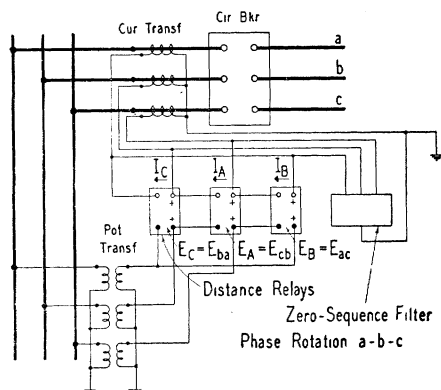


Figure 12 (above). Distance relay connections for line-fault protection using delta-connected relays and zero-sequence filter

Figure 13 (below). Distance relay connections for single line-to-ground faults using current compensation

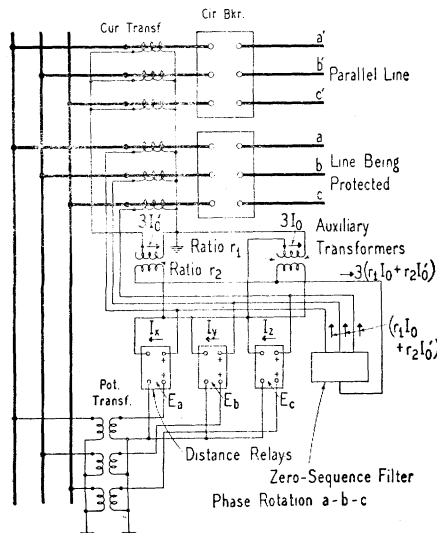
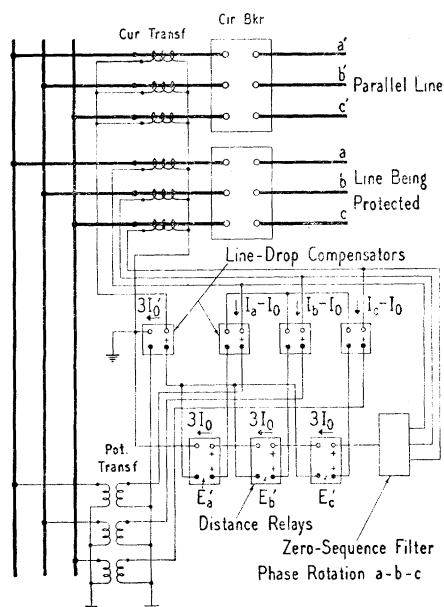


Figure 14. Distance relay connections for single line-to-ground faults using voltage compensation



system external to the protected section.

Two methods are available for obtaining the proper current. The necessary current transformer connections are shown in Figures 7 and 8. The first method consists of connecting the current transformers in delta and the relays in Y, in accordance with Figure 7. Connections for a set of three relays using this connection are shown in Figure 11. The second method consists of connecting the current transformers in Y and the relays in delta, in accordance with Figure 8. Connections for three relays using this connection are shown in Figure 12. In this case, the currents through the relays are I_A , I_B , and I_C , which are equal to one third of the currents obtained by the method of Figure 7. Hence the impedance indicated by the relay is three times as great, other conditions remaining the same, as it is using the delta-connected current transformer scheme. When the system has a grounded neutral, a zero-sequence filter is required with the connection of Figure 12 to provide a path for the zero-sequence or residual current. One form of such a filter is shown in Figure 8. Any grounding transformer connection is suitable for this purpose, so that several other designs are possible.

Ground Faults—

Conventional Distance Relaying

It generally is recognized that the high value of resistance which may be present in ground faults is sufficient to eliminate the ordinary impedance relay from consideration. However, the use of a relay which effectively eliminates the resistance component often is considered, and an analysis of the available currents and voltages during ground faults is, therefore, pertinent. It is customary to use the line-to-ground potential and line current for energizing the relay, although use of the residual current has been suggested. In setting up the conditions for this type of fault in Appendix VI, the fault is assumed to occur on conductor a . The line-to-neutral voltage on the faulted phase is, therefore, E_{ag} . Equation 159 of Appendix VI gives the value of E_{ag} during this type of fault. Using the line current, I_{agF} , for tripping, the indicated impedance is

$$Z_R = mZ_{IGH} + R_G + \frac{I_{aHF}}{I_{agF}}R_G + \frac{I_{0GF}}{I_{agF}}m(Z_{0GH} - Z_{1GH}) + \frac{I_{0GH}}{I_{agF}}mM_{0GH} \quad (20)$$

It is evident that this quantity is af-

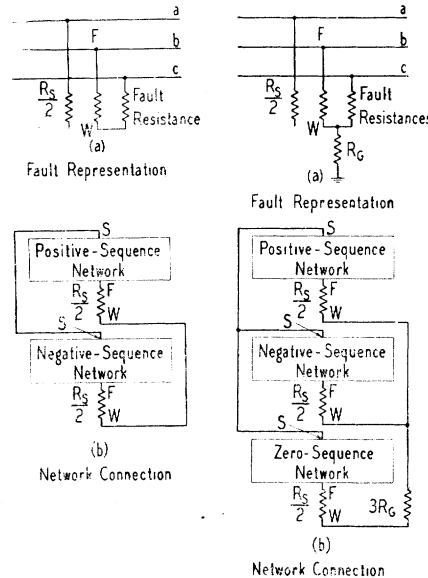


Figure 15 (above, left). Representation of line-to-line faults

Figure 16 (above, right). Representation of double line-to-ground faults

ected by the fault resistance, the residual current in the line being protected, and by the residual current in the parallel line.

Where two lines parallel each other at close spacings, the zero-sequence mutual impedance between the lines becomes quite appreciable in comparison with the self-impedances, and, when m is large, that is, when a fault occurs near the remote end of the section, the value of I_{0GH}/I_{agF} becomes of the order of one third on a normally designed system, so that an important term is added. However, if the parallel line is not in service, this term is not present, and considerable variation in the indicated impedance is possible. Of course, if only a single line is involved, this term will not enter in any case.

The variable term $(I_{0GF}/I_{agF})m \times (Z_{0GH} - Z_{1GH})$ is also a source of trouble, as the ratio (I_{0GF}/I_{agF}) is not fixed by conditions within the section itself, unless there is only one path for the fault current to take, true in this case only after the circuit breaker at H has opened. In general, where the fault is near the remote end of the section, the ratio (I_{0GF}/I_{agF}) often will be less than one third when the circuit breaker at H is closed, and equal to one third when the circuit breaker at H is open, causing an increase in impedance due to the opening of the circuit breaker, at which time a decrease is much to be preferred if any change must occur. The value one third given for the ratio (I_{0GF}/I_{agF}) is imposed by the conditions at the fault when the fault

current comes entirely from one point. When the fault current divides and comes from two or more points to the fault, the proportion of the current in the conductor which is residual or zero sequence depends upon the relative impedance to phase and ground currents in that branch, and, due to the fact that the impedance of the transmission line is much higher to ground current than to phase current, a relatively greater portion of the residual current usually will come from the end of the line nearer the fault.

The variations in indicated vector impedance discussed affect the indicated reactance in much the same proportions, so that the use of a reactance relay presents no advantages when considered from the standpoint of these errors alone, and the justification for such a relay rests on the fact that it reduces the effect of the two resistance terms.

Use of the residual current does not give any improvement over the use of line current, as the voltage E_{ag} is not directly proportional to the residual current, even when fault resistance is neglected, and a detailed analysis of this method therefore is not included.

Ground Faults—

Improved Distance Relaying

From what has preceded it is evident that the use of either impedance or reactance relays in the manner suggested will not be entirely satisfactory, and methods for improving the performance are necessary if the distance principle is to find extensive application for ground faults. As the currents and voltages which are available at the relaying station do not give satisfactory indications as they stand, it may happen that they may be modified in accordance with known characteristics of the circuits to give more suitable quantities. Modification may be made in either the current, or the voltage, or both. In any case, much improved performance may be obtained.

CURRENT COMPENSATION

Equation 159, giving E_{ag} during this type of fault, may be written in a different form by factoring mZ_{1GH} out of all terms except those involving R_G . We obtain

$$E_{ag} = \left(I_{agF} + I_{0GF} \frac{(Z_{0GH} - Z_{1GH})}{Z_{1GH}} + I_{0GH} \frac{M_{0GH}}{Z_{1GH}} \right) mZ_{1GH} + (I_{agF} + I_{aHF})R_G \quad (21)$$

If now we are able to use for tripping a current I_r defined by the equation

$$I_r = I_{aGF} + I_{0GF} \frac{Z_{0GH} - Z_{1GH}}{Z_{1GH}} + I_{0GH} \frac{M_{0GH}}{Z_{1GH}} \quad (22)$$

we obtain a very simple result, as follows

$$Z_R = m Z_{1GH} + \frac{I_{aGF} + I_{aHF}}{I_r} R_G \quad (23)$$

The first term of this expression is directly proportional to the distance from the fault to the relay and is independent of conditions external to the section being protected. The second term is a function of the fault resistance and has the effect of a pure resistance if $(I_{aGF} + I_{aHF})$ is in phase with I_r . As, in general, all the impedances involved in equation 22 will have nearly the same phase angle, unless the line resistances are very large, and also, as I_{aGF} , I_{0GF} , and I_{0GH} are usually nearly in phase for this type of fault, this will be nearly true. In general, the variations in phase will be of the same order as those occurring between corresponding currents in the preceding analysis of line faults. Hence, for faults having fault resistances in the same proportions, just as satisfactory protection can be obtained for ground faults as for line faults, using distance relays. However, the fault resistance generally reaches considerably higher values, so that the variations will be somewhat larger than the minimum obtainable on line faults, and a relay which is substantially unaffected by fault resistance generally will be required.

There are several methods of obtaining the current I_r for relaying purposes. Where the line resistance is not too large, it will be satisfactory to assume that $(Z_{0GH} - Z_{1GH})$, M_{0GH} , and Z_{1GH} are all in phase. If this is true, the ratios of these quantities, appearing in equation 22 for I_r , become pure numbers or transformer ratios, so that these currents may be added to the relay currents by means of transformers of corresponding ratio energized from the residual currents in the two lines respectively. For this purpose, the relays may be provided with additional current coils which carry the added currents. Another method is to introduce these currents directly into the normal coil by means of the zero-sequence filter, according to the scheme of Figure 9. The connections for a set of relays using this method are shown in Figure 13. The currents flowing through the relays become I_x , I_y , and I_z , given by equations 62, 63, and 64 in which $r_1 = |(Z_{0GH} - Z_{1GH})|/Z_{1GH}$ and $r_2 = M_{0GH}/Z_{1GH}$.

Where the resistances in the lines are large, correction for phase-angle differences in the impedances may be required.

Various means of making this correction are possible, but they introduce considerable complication and are not considered here.

VOLTAGE COMPENSATION

It has been stated earlier in this paper that it is desirable to obtain a constant relay indication for all faults in a particular location and that this indication should be proportional to the distance from the relay to the fault. This is a sufficient condition to establish the balance points accurately and divide the faults into the proper classes, but with relays operating on the balance-point principle it is not a necessary condition. It is, however, necessary for the relays to receive a constant indication for faults at the chosen balance points. For faults at other locations, it is immaterial whether the indication is proportional to the distance from the relay to the fault or not, provided the indication given does not exceed the limits provided by faults at the balance points. This is due to the fact that a distance relay of the balance-point type will trip with practically a constant speed for all faults within the limits of the class in which the fault falls, regardless of whether the indicated value is well within the class or near the limits.

Use is made of this principle in applying voltage compensation to distance ground relays. In analyzing this application, the relaying, voltage E_{aG} may be expressed more conveniently in the form

given by equation 167 in Appendix VI as

$$E_{aG} = (I_{1GF} + I_{2GF})mZ_{1GH} + I_{0GF}mZ_{0GH} + I_{0GH}mM_{0GH} + 3(I_{0GF} + I_{0HF})R_G$$

If I_{0GF} is used for energizing the relay, considerable error and variation will be produced by the terms not proportional to I_{0GF} . However, for faults at particular locations, these terms, other than the resistance term, may be removed by means of line drop compensators. Let us assume that the fractional distance to the first balance point is n . Then we may subtract from the relay voltage the quantity

$$(I_{1GF} + I_{2GF})nZ_{1GH} + I_{0GH}nM_{0GH}$$

If a fault occurs at the balance point, $m = n$, and we have

$$E_{aG} = I_{0GF}nZ_{0GH} + 3(I_{0GF} + I_{0HF})R_G \quad (24)$$

If I_{0GF} is used for relaying current, the measured impedance becomes

$$Z_R = n Z_{0GH} + 3R_G + 3 \frac{I_{0HF}}{I_{0GF}} R_G \quad (25)$$

This value of Z_R is composed of a definite impedance $n Z_{0GH}$, a pure resistance $3R_G$, and a term which may have a reactance component unless I_{0HF} is exactly in phase with I_{0GF} . The factor 3 enters here in connection with the resistance, making the apparent resistance much larger, but the impedance term in this case is Z_{0GH} , which is usually three or more times the positive-sequence impedance for the line, Z_{1GH} , so that the ratio of resistance to reactance in Z_R for a given value of R_G is not increased, over that obtained by current compensation, and, therefore, the errors introduced by the difference in phase of I_{0HF} and I_{0GF} will be of about the same percentage as before.

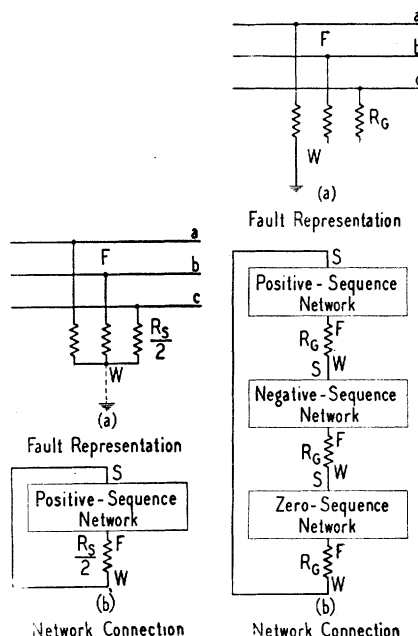
If the fault occurs at some other location, m is no longer equal to n , and Z_R becomes

$$Z_R = mZ_{0GH} + 3R_G + 3 \frac{I_{0HF}}{I_{0GF}} R_G + \frac{I_{1GF} + I_{2GF}}{I_{0GF}} (m - n)Z_{1GH} + \frac{I_{0GH}}{I_{0GF}} (m - n)M_{0GH} \quad (26)$$

The two additional terms are positive for m greater than n and negative for m less than n , so that, for faults beyond n , Z_R increases by a greater percentage than m does, and for faults nearer than n , Z_R decreases by a greater percentage than m . Hence, as m becomes quite small, the impedance may become negative. As the reactance elements of most reactance relays will trip for negative reactance, this introduces no difficulty so long as the

Figure 17 (below, left). Representation of 3-phase faults

Figure 18 (below, right). Representation of single line-to-ground faults



directional element is not energized from the compensated voltage and operates properly.

With the foregoing scheme, four compensators are required per group of three ground relays. Three of these compensators must be energized from the positive- and negative-sequence components of the current in the corresponding line conductors and the remaining compensator from the residual current in the parallel line. The current ($I_{1GF} + I_{2GF}$) is equal to ($I_{0GF} - I_{0HF}$) or in other words the required current may be obtained by removing the zero-sequence component from the line current. This may be accomplished by means of the zero-sequence filter previously described. Use of this method of compensation is made in the relay connections shown in Figure 14. However, the current through each relay is ($3 I_0$) so that the indicated impedances are one third of the values given by equations 25 and 26.

Other schemes of compensation may be utilized, making use of line current, residual current or any of the symmetrical components of the line current for tripping the relay, and eliminating the terms in the voltage not proportional to the distance from the fault by means of compensators.

Operation of Ground Relays on Line Faults

Operation of ground relays upon the occurrence of line-to-line faults is not to be expected, unless the fault current is very large, because the line-to-ground voltage maintains a large proportion of its normal value, and the indicated impedance or reactance is, therefore, usually sufficiently great to prevent operation. In the few cases where the line-to-line fault currents are large enough to operate the ground relays, it is almost certain that the corresponding line relay also should operate, so that false tripping will not occur, but indication of a more severe fault involving ground occasionally may be obtained. The indicated impedance or reactance is not fixed by the impedance of the line and fault, but is affected by the remainder of the system, so that determination of the actual value requires consideration of each individual case.

Double line-to-ground and 3-phase faults previously have been classed as line faults. However, operation of the ground relays upon the occurrence of such faults normally will take place, and an analysis of the operation is required. Analysis is given only for ground relays provided with compensation as described.

DOUBLE LINE-TO-GROUND FAULTS

The fault is assumed to occur between conductors b and c and ground. Hence, the relaying voltages are E_{bG} and E_{cG} , given by equations 122 and 123. Factoring out mZ_{1GH} , as was done during the analysis of current compensation for single line-to-ground faults, we obtain

$$E_{bG} = \left[I_{bGF} + I_{0GF} \frac{(Z_{0GH} - Z_{1GH})}{Z_{1GH}} + I_{0GH} \frac{M_{0GH}}{Z_{1GH}} \right] mZ_{1GH} + (I_{bGF} + I_{bHF}) \frac{R_S}{2} + (I_{0GF} + I_{0HF}) 3R_G \quad (27)$$

$$E_{cG} = \left[I_{cGF} + I_{0GF} \frac{(Z_{0GH} - Z_{1GH})}{Z_{1GH}} + I_{0GH} \frac{M_{0GH}}{Z_{1GH}} \right] mZ_{1GH} + (I_{cGF} + I_{cHF}) \frac{R_S}{2} + (I_{0GF} + I_{0HF}) 3R_G \quad (28)$$

Except for the fault resistance terms, the foregoing equations are of exactly the same form as would be obtained for single line-to-ground faults on the corresponding phases. Hence, exclusive of the fault-resistance terms, if the relaying currents are equal to the expressions in brackets, the same indicated impedance is obtained for double line-to-ground faults as for single line-to-ground faults. If the circuit impedances are such that we may assume $(Z_{0GH} - Z_{1GH})$ and M_{0GH} in phase with Z_{1GH} , these currents reduce to I_{yGF} and I_{zGF} , as given by the connections of Figure 13 and equations 63 and 64, r_1 and r_2 being $|(Z_{0GH} - Z_{1GH})|/Z_{1GH}$ and M_{0GH}/Z_{1GH} respectively. Under these assumptions Z_R for the two relays becomes

$$Z_{R(b)} = mZ_{1GH} + \frac{I_{bGF} + I_{bHF}}{I_{yGF}} \frac{R_S}{2} + \frac{I_{0GF} + I_{0HF}}{I_{yGF}} 3R_G \quad (29)$$

$$Z_{R(c)} = mZ_{1GH} + \frac{I_{cGF} + I_{cHF}}{I_{zGF}} \frac{R_S}{2} + \frac{I_{0GF} + I_{0HF}}{I_{zGF}} 3R_G \quad (30)$$

For this type of fault, the currents in the line conductors are usually considerably out of phase with the residual currents, and the relay currents, being made up of both, will be intermediate in phase between the two. Hence the individual resistance terms never will appear as pure resistance but will have reactance components. However, as the relaying current has a phase position intermediate between the line and residual currents, it will lead one and lag the other, so that the

reactance effects will tend to compensate. Therefore, the amount of variation introduced into the indicated reactance usually will be small, unless the two resistances R_S and R_G run to opposite extremes or are large in comparison with the reactance of the line.

A similar analysis may be given for voltage compensation. The line-to-neutral voltages expressed in the most convenient form are given by equations 132 and 133. With the scheme of compensation shown by Figure 14, the voltages subtracted from the line-to-neutral voltages will be $(a^2 I_{1GF} + a I_{2GF}) n Z_{1GH} + I_{0GH} n M_{0GH}$ and $(a I_{1GF} + a^2 I_{2GF}) n \times Z_{1GH} + I_{0GH} n M_{0GH}$ respectively.

The resultant relay voltages become

$$E_{bG} = I_{0GF}(mZ_{0GH} + 3R_G) + 3I_{0HF}R_G + (I_{bGF} + I_{bHF}) \frac{R_S}{2} + (a^2 I_{1GF} + a I_{2GF}) \times (m-n)Z_{1GH} + I_{0GH}(m-n)M_{0GH} \quad (31)$$

$$E_{cG} = I_{0GF}(mZ_{0GH} + 3R_G) + 3I_{0HF}R_G + (I_{cGF} + I_{cHF}) \frac{R_S}{2} + (a I_{1GF} + a^2 I_{2GF}) \times (m-n)Z_{1GH} + I_{0GH}(m-n)M_{0GH} \quad (32)$$

As $3I_{0GF}$ is the relaying current, the indicated impedances become

$$Z_{R(b)} = \frac{mZ_{0GH}}{3} + R_G + \frac{I_{0HF}}{I_{0GF}} R_G + \frac{I_{bGF} + I_{bHF}}{3I_{0GF}} \frac{R_S}{2} + \frac{a^2 I_{1GF} + a I_{2GF}}{3I_{0GF}} \times (m-n)Z_{1GH} + \frac{I_{0GH}}{3I_{0GF}} (m-n)M_{0GH} \quad (33)$$

$$Z_{R(c)} = \frac{mZ_{0GH}}{3} + R_G + \frac{I_{0HF}}{I_{0GF}} R_G + \frac{I_{cGF} + I_{cHF}}{3I_{0GF}} \frac{R_S}{2} + \frac{a I_{1GF} + a^2 I_{2GF}}{3I_{0GF}} \times (m-n)Z_{1GH} + \frac{I_{0GH}}{3I_{0GF}} (m-n)M_{0GH} \quad (34)$$

Here again, exclusive of resistance terms, for a double line-to-ground fault at the balance point, that is, for $m=n$, the indicated impedances are the same as would be obtained for a single line-to-ground fault on the same conductors. However, the resistance terms involving $R_S/2$ may be a source of considerable error whenever R_S becomes large in comparison with Z_{0GH} , as $(I_{bGF} + I_{bHF})$ and $(I_{cGF} + I_{cHF})$ will be considerably out of phase with I_{0GF} , thereby producing an apparent reactance which will be positive in one phase and negative in the other.

THREE-PHASE FAULTS

On 3-phase faults, using current compensation, the compensated relaying

currents do not differ from the phase currents, as no ground or zero-sequence current flows. For the same reason the zero-sequence impedance does not appear, so that the indicated impedance on phase *a* will be

$$Z_R = mZ_{IGH} + \frac{R_S}{2} + \frac{I_{aHF} R_S}{I_{aGF} 2} \quad (35)$$

with a similar expression for each of the other phases.

Using the scheme of voltage compensation described, the relays will not be operated, as the zero-sequence current is used, and this current is zero for a balanced 3-phase fault.

From the foregoing it appears that, fundamentally, the proposed schemes of ground relaying are not affected adversely by line faults, if the fault resistances are relatively small or bear certain relations to each other, but, if the fault resistances are large, the possibility of considerable variations appears during double line-to-ground faults, thus requiring larger margins in setting or introducing the possibility of false tripping. On relatively short lines, where the fault resistance may become large in comparison with the line impedance, it may be desirable to lock out the ground relays upon the occurrence of double line-to-ground or 3-phase faults, this being accomplished satisfactorily in most cases by the use of simple voltage relays operating on line-to-ground voltage, locking out the ground relays when low voltage occurs on any two phases.

Operation of Line Relays on Ground Faults

Operation of line relays upon the occurrence of ground faults is very improbable as the line-to-line voltage is affected but slightly by a single line-to-ground fault. Here again the values indicated by the line relays are not fixed definitely by the fault and section impedances but depend upon the remainder of the system as well, so that the values must be determined individually in each case, whenever this question must be answered

Effect of Loads

In the preceding discussion and analysis, the system load has been neglected. However, the equations in the appendixes for voltages and currents at the fault, and at a relaying point on the faulted line, have been derived in a perfectly general manner from the conditions at the fault, and are independent of the network external to the two lines between *G* and *H*.

As it is assumed that no loads will be connected to the lines between *G* and *H*, the equations are valid whether loads are included or not. However, the loads will affect, to some extent, the current and voltage distribution in the network and, therefore, the actual values of the voltages and currents entering the equations. The expressions for impedance on the faulted phase do not involve explicitly any voltage expressions but do involve certain current ratios, and these ratios will be affected somewhat by the loads. If the delta-current relaying methods are used for line faults and the current-compensation method for ground faults, as previously described, these current ratios enter only in the fault-resistance terms and have no effect if the fault resistances are zero. If the voltage-compensation method is used for ground faults, the same is true for faults at the balance point, and it is unlikely that the effect of load will cause any serious difficulty for faults that are not close to a balance point. Hence, with these methods of relaying, the system load affects the relaying indications only in so far as the redistribution of fault current changes the effect of fault resistance.

Summary

In the preceding investigation the quantities upon which distance relays must depend for their operation have been determined for all possible types of faults occurring at a single location. It has been shown that the quantities obtained by the use of earlier distance relay connections for both line and ground faults do not give the closest approach to the desired result, a constant indication for all types of faults at a given location. It has been shown further that, by the use of different connections and certain auxiliary apparatus, a constant relaying indication may be obtained by the line relays for all types of line faults and by the ground relays for all types of faults involving ground, if the fault resistance is zero, and, if the fault resistance is not zero, that smaller deviations from the normal value will be obtained for a given fault resistance. The analysis has been based upon the assumptions of sine wave currents and voltages, constant circuit impedances, and negligible distributed capacitance in the faulted sections of line. It has been assumed further that the fault impedance is resistance only and that it is effectively constant throughout each cycle. All of these assumptions, except perhaps the latter, are reasonably valid as far as relaying is concerned, al-

though some consideration must be given to the d-c component of the short-circuit currents with very high-speed relays. Sufficient data are not yet available to state the characteristics of fault impedances definitely, but undoubtedly further information will be obtained from future investigations. Until more accurate data are available, certain margins determined by experience must be provided in relay settings to allow for deviations from the assumptions.

If the suggested connections are used, the vector impedance which is indicated by the voltage and current applied to the relay on the faulted phase, at least for all faults near the first balance point, will consist essentially of three terms, when the fault is fed from both ends of the section. They are: a circuit impedance term which is proportional to the distance from the relay to the fault, a fault resistance term, and a term proportional to the fault resistance multiplied by the vector ratio of two currents. An impedance relay operates upon the magnitude of the vector sum of the three terms, and, as the two resistance terms are nearly at right angles to the impedance term, the magnitude of the sum will be greater than the magnitude of the circuit-impedance term, although the increase will be much less than the added resistance terms, unless the latter are very large. To maintain a balance at the relay, the circuit-reactance term must decrease as the fault-resistance terms increase, so that the effect of fault resistance is to shift the balance point toward the relay. A reactance relay, on the other hand, operates on the reactive component of the three terms, and, therefore, is unaffected by the first fault-resistance term. However, the second resistance term will have an apparent reactance if the two currents involved in the ratio are out of phase, and this apparent reactance may be positive or negative, depending upon which current leads in phase. Hence the balance point may be shifted in either direction by the presence of fault resistance. Although the actual shift usually will be less with a reactance relay, the fact that it may be in either direction must be taken into account, so that where the fault resistances are moderate in comparison with the reactance of the section, the impedance relay usually will give as satisfactory protection as a reactance relay. However, where the fault resistances are larger, as on ground faults, the increases in impedance become so great that the ordinary impedance relay is disqualified.

In the preceding analysis, all current

and voltage transformer ratios have been assumed as unity. In setting the relays, the current and voltage transformer ratios must be taken into account. It is customary also to set the relays under the assumption of zero fault resistance, choosing a balance point under this condition which gives a suitable margin for the variation introduced by fault resistance. If n is the fraction of the section from the relay to the balance point, r_v the ratio of the circuit voltage, line-to-line for line faults, line-to-ground for ground faults, to the voltage impressed on the relay circuit, and r_c the current transformer ratio, the actual settings of the relays will be as given below for the different suggested connections. If the voltage is obtained from high-voltage potential transformers, as shown in the diagrams, r_v is simply the potential transformer ratio. For line relaying using delta-connected current transformers (Figure 11), and for ground relaying using current compensation (Figure 13), the relay should be adjusted to balance for an impedance

$$Z_R = nZ_{IGH}r_c/r_v \quad (36)$$

For line relaying using delta-connected relays (Figure 12), the relays should balance for an impedance

$$Z_R = 3nZ_{IGH}r_c/r_v \quad (37)$$

For ground relaying using the suggested scheme of voltage compensation, the relays should balance for an impedance

$$Z_R = nZ_{0GH}r_c/(3r_v) \quad (38)$$

The settings for both balance points of a 3-zone relay will be made in the same manner, a different value of n being used in the two cases.

Application

Distance relaying for protection against line and ground faults is suitable for general application to either single- or multiple-circuit lines, the only theoretical limitation being the variations in the balance-point locations introduced by the effects of fault resistance. Either the first or second balance point may be the limiting factor. The first balance point always must remain in the protected section, and the second balance point in the adjacent section. Furthermore, for general application the second balance point never must extend beyond the first balance point of the relay at the next sectionalizing point, or both circuit breakers will be tripped for some faults. The shorter the line sections become, the more difficult these conditions are to fulfill, so that a certain minimum length of

section or impedance is required. This minimum depends upon such factors as short-circuit current distribution and the fault resistances encountered, so that it cannot be fixed definitely.

Because of the higher values of fault resistance generally met during ground faults, distance ground relaying cannot be applied to lines as short as the minimum which can be protected satisfactorily by distance line relaying. However, where the lines are sufficiently long, this type of protection may offer substantial advantages. High-speed ground relaying of single lines is difficult by the customary overcurrent methods, as the ground current may vary over wide limits for a fault in a given location, thus requiring selective time settings to obtain discrimination. Either compensated current or compensated voltage methods of distance relaying appear to be well suited for this purpose.

The current compensation method also appears to be well adapted to cases where grounded-neutral power transformers are made an integral part of the line and the high-voltage circuit breakers are omitted. This case is particularly difficult to relay by ordinary methods of ground-fault protection, as, after the low-voltage circuit breaker at one end is open because of a fault near that end, most of the ground fault current continues to flow from the grounded-neutral transformer at that end, making it very difficult to open the circuit breaker at the opposite end. As the compensated distance relay measures the impedance or reactance from the relay to the fault, independent of the relative distribution of line and ground current, it is essentially unaffected by such system connections. The particular compensated voltage method described is not well suited to this application because of the low ground current available for operating the relays at the end remote from the fault, but a compensated voltage method using line current for relay operation instead of ground current, also would give good results.

It should be pointed out that this analysis has considered only the possible accuracy, and has not taken into account the performance of the relay itself, its design, its simplicity, its reliability, or its speed of operation. All of these factors are important and must be taken into account in a final choice of the type of distance relay to be used for a given application.

The equations derived in the appendixes giving the voltages and currents at relaying points in terms of circuit impedances and conditions at the fault have

been used in the paper only for the purpose of determining the performance of distance relays. However, they may be found useful for many other purposes and constitute a contribution to the further development of the method of symmetrical components.

Conclusions

This analysis has shown:

1. That the earlier conventional methods of distance relaying are subject to errors other than those caused by fault impedance.
2. That these errors may be eliminated in relays protecting against line faults by the use of "delta relaying." Delta relaying may be obtained either by connecting the current transformers in delta or by connecting the current windings of the relays in delta, using a suitable zero-sequence filter.
3. That these errors may be eliminated in relays protecting against ground faults by the use of current or voltage compensation and, therefore, that distance relaying is suitable for ground fault protection, if the impedance of the section being protected is sufficiently great.
4. That the limitation in the application of distance relaying is the error introduced by the presence of fault impedance, which will decrease with increasing section impedance, and that this limitation affects distance relays of both the impedance and reactance types, although to a different extent.
5. That distance relaying may be made to approach even closer to the desired relaying ideal and that within the limits of its application it should give satisfactory protection, regardless of generator capacity or system conditions.

Appendix I. Notation

- $\alpha = e^{j120} = -0.5 + j0.866$
 F = physical point of fault along transmission line
 G = substation where relaying is considered
 H = substation at opposite end of line section
 P, Q = equivalent junction points in zero-sequence network
 S = equivalent point of zero potential
 T = equivalent point of application of E_{1T}
 U = equivalent point of application of E_{1U}
 W = equivalent point of fault when fault resistance is present
 E_{1T} = equivalent positive-sequence generated voltage of part of system feeding station G
 E_{1U} = equivalent positive-sequence generated voltage of part of system feeding station H
 E = voltage symbol, with subscript notation as follows:
 First subscript:
 a, b , or c = line-to-neutral voltage of corresponding phase
 A, B , or C = line-to-line voltage of corre-

sponding phases, *cb*, *ac*, *ba* respectively. See Figure 3

1, 2, 0=symmetrical components of line-to-neutral voltage, phase *a* as reference
Second subscript:

F, *G*, *H*, *W*=point at which voltage is measured

I=current symbol, with subscript notation as follows:

First subscript:

a, *b*, or *c*=line-to-neutral current in corresponding line

A, *B*, or *C*=line-to-line or delta current of corresponding delta phase, obtained by delta connection of relays. See Figure 8.

α , β , or γ =delta current obtained from delta-connected current transformers. See Figure 7

x, *y*, *z*=relaying currents obtained from the connection of Figure 9

1, 2, 0=symmetrical components of line-to-neutral current, phase *a* as reference
Second and third subscripts:

GF, *HF*, *GH*, *TG*, *UH*, *SG*, *SH*, *FW*, and so forth=stations between which current flows. Assumed positive direction of current is from station of first letter to that of second.

Z=impedance symbol, scalar value, with subscript notation same as for *Z*

Z=impedance symbol, vector value, with subscript notation as follows:

First subscripts, if only one:

R=impedance indicated by relay for various types of faults

D=impedance of current winding of relays or other current devices

First subscript, if more than one:

1, 2, 0=sequence of current with which impedance is to be used

Second and third subscripts:

GF, *HF*, *GH*, *TG*, *UH*, *SG*, *SH*=stations between which impedance is measured. Between stations *G* and *H* value is for one line only

*M*_{0GH}=zero-sequence mutual impedance between the two parallel lines from *G* to *H*

*R*_s=fault resistance for line faults between two line conductors

*R*_g=fault resistance for ground faults

m=fraction of the distance from *G* to *H* at which fault is assumed to occur

n=fraction of the distance from *G* to *H* at which the first balance point is located

*r*_v=potential transformer voltage ratio

*r*_c=current transformer current ratio

*r*₁, *r*₂=auxiliary transformer turn ratios, See Figures 9 and 13

Appendix II. Network Representation and Fault Currents

In order to calculate the fault currents and voltages during unbalanced faults, by the method of symmetrical components, it is necessary to set up sequence networks for the flow of positive-, negative-, and zero-sequence currents, corresponding to the system under investigation. The positive- and negative-sequence networks comprise a group of impedances having a branch corresponding to every branch of the single-line diagram. Synchronous machines are represented by their equivalent circuits for a single phase, with single-phase voltages inserted at the proper point. As synchronous machines generate only positive-sequence voltages, generated voltages appear only in the positive-sequence network.

The positive- and negative-sequence networks, for the system shown in Figure 2, are given in Figures 4 and 5, respectively. In the positive-sequence network, two voltages, *E*_{1T} and *E*_{1U}, are inserted to represent the voltages generated by the equivalent synchronous machines at *T* and *U* respectively. The impedance of each branch of the networks is represented by the symbol *Z* with three subscripts, the first a number indicating the sequence network in which the impedance appears, and the second and third the points between which the impedance is measured. As *E*_{1T} and *E*_{1U} are expressed in terms of line-to-neutral voltage, the other terminals of these voltages are connected to the common neutral *S* of the positive-sequence network. In the negative-sequence network, these voltages are equal to zero, so that the terminals *T* and *U* may be omitted and the impedances connected directly to the corresponding point *S*. For all static apparatus the negative-sequence impedance is equal to the positive, so that the branches between *G* and *H*, corresponding to the transmission line, will have the same impedance as in the positive-sequence network. This has been represented by substituting *Z*_{1GH} for *Z*_{2GH} in Figure 5. The fault is assumed to occur at *F*, intermediate between *G* and *H*, at a fraction *m* of the distance from *G* to *H*. As the impedance from *G* to *H* of one line has been taken as a unit, and as, neglecting distributed capacitance, the impedance is proportional to the distance, the impedances from *G* to *F* become *mZ*_{1GH} and from *F* to *H* become (1-*m*)*Z*_{1GH}. As the branches between *T* and *G* and between *H* and *U* contain synchronous-machine impedances, they will in general be different for the different sequences.

The zero-sequence network is made up only of those branches which may carry current having an earth-return path, and, therefore, in general it has a smaller number of branches. The corresponding impedances are also usually quite different and generally larger, although in certain special cases such as grounding transformers, the zero-sequence impedance to ground may be less than the corresponding positive- and negative-sequence impedances. The zero-sequence network for the system shown in Figure 2 is given in Figure 6. As adequate grounding has been assumed at each end of the system, branches are required from *G* and *H* to *S*. In the positive- and negative-sequence networks, the mutual impedance between two parallel lines is generally small

enough to be neglected, but in the zero-sequence network this is not true, because of the great distance between the conductors and the ground return, as compared with the spacing between the two parallel lines. Because of the common use of the earth return and ground wires, the mutual impedance also may contain a resistance component. The mutual impedance is represented by the vector symbol *M*_{0GH}.

It is well known that two circuits, having a mutual impedance between them and connected together at one end, may be replaced by three impedances connected in star, the branch connected to the original junction being equal to the mutual impedance and the other branches being equal to the original branches minus the mutual impedance. Use is made of this property in setting up the zero-sequence impedances between stations *G* and *H* and the fault *F*. The symbols for impedance are similar to those used in the positive- and negative-sequence networks, except that the first subscript is now 0 instead of 1 or 2.

The current flowing in the various branches of the networks is indicated by the symbol *I* with three subscripts, the first a number (1, 2, or 0) indicating the sequence of the current, and the second and third letters indicating the stations between which the current is flowing, the assumed positive direction being from the station indicated by the first letter toward the one indicated by the final letter.

If the currents and voltages at various points in the sequence networks are known, the actual currents and voltages in the three phases at corresponding points in the actual system may be found by combining the currents and voltages according to relations that follow. Phase *a*, line-to-neutral, has been assumed as reference, and the phase rotation is assumed to be *a*, *b*, *c*. Then

$$E_a = E_1 + E_2 + E_0 \quad (39)$$

$$E_b = \alpha^2 E_1 + \alpha E_2 + E_0 \quad (40)$$

$$E_c = \alpha E_1 + \alpha^2 E_2 + E_0 \quad (41)$$

and

$$I_a = I_1 + I_2 + I_0 \quad (42)$$

$$I_b = \alpha^2 I_1 + \alpha I_2 + I_0 \quad (43)$$

$$I_c = \alpha I_1 + \alpha^2 I_2 + I_0 \quad (44)$$

The voltages between lines, or delta voltages, defined in accordance with the vector diagram of Figure 3, may be determined by subtracting the phase voltages in pairs, as follows. In performing the subtraction it is convenient to make use of the following relations concerning the vector α

$$1 - \alpha = j\sqrt{3}\alpha^2 \quad (45)$$

$$\alpha - \alpha^2 = j\sqrt{3} \quad (46)$$

$$\alpha^2 - 1 = j\sqrt{3}\alpha \quad (47)$$

Thus:

$$E_A = E_c - E_b = j\sqrt{3}(E_1 - E_2) \quad (48)$$

$$E_B = E_a - E_c = j\sqrt{3}(\alpha^2 E_1 - \alpha E_2) \quad (49)$$

$$E_C = E_b - E_a = j\sqrt{3}(\alpha E_1 - \alpha^2 E_2) \quad (50)$$

For the purposes of relay operation, it will be desirable to consider three special forms of current transformer connection,

as shown in Figures 7, 8, and 9. In the following analysis it will be assumed that the main current transformer ratio is 1-to-1, so that the results will be the same as if the actual line currents had been manipulated. In actual cases the currents in the secondary circuits can be obtained by dividing by the current transformer ratio. In Figure 7 the current transformers are connected in delta, and leads from the corners of the delta are taken to current utilizing devices, connected in star. With this connection the leads carry currents which are the differences of the line currents taken in pairs. It will be noted that these currents are of the same mathematical form as the delta voltages E_A , E_B , and E_C , and the corresponding currents are designated by I_α , I_β , and I_γ , determined as follows

$$I_\alpha = I_c - I_b = j\sqrt{3}(I_1 - I_2) \quad (51)$$

$$I_\beta = I_a - I_c = j\sqrt{3}(a^2 I_1 - a I_2) \quad (52)$$

$$I_\gamma = I_b - I_a = j\sqrt{3}(a I_1 - a^2 I_2) \quad (53)$$

In Figure 8, the utilization devices are assumed to have identical impedances, Z_D , and are connected in delta. At the corners of the delta, the line currents I_a , I_b , and I_c are introduced. This delta connection offers no path to any residual or zero-sequence current in the line conductors, so that, if the system is grounded, allowing zero-sequence current to flow in the line, some additional path must be provided for this current. It is also desirable that this additional path pass as little positive- and negative-sequence current as possible. These conditions are satisfied by means of the small interconnected-star transformer bank shown. Any other grounding transformer connection may be used as well for this purpose, as it is the function of a grounding transformer to offer very high impedance to all but zero-sequence currents.

The currents in the branches of the delta may be denoted by the symbols I_A , I_B , and I_C . Following the convention previously adopted, I_A is the current in the delta branch opposite the line a , and so forth. Writing the equation for the voltage drop around the delta, we obtain

$$(I_A + I_B + I_C)Z_D = 0 \quad (54)$$

or

$$I_A + I_B + I_C = 0 \quad (55)$$

The interconnected-star transformer or, more generally, the zero-sequence filter bypasses all of the zero-sequence currents and, if properly designed, only a negligible portion of the positive- and negative-sequence currents, so that at the junction of the delta we may write

$$I_B - I_C = I_a - I_0 = I_1 + I_2 \quad (56)$$

$$I_C - I_A = I_b - I_0 = a^2 I_1 + a I_2 \quad (57)$$

$$I_A - I_B = I_c - I_0 = a I_1 + a^2 I_2 \quad (58)$$

These four equations, of which only three are independent, may be solved for I_A , I_B , and I_C , giving

$$I_A = j\frac{\sqrt{3}}{3}(I_1 - I_2) = \frac{I_\alpha}{3} \quad (59)$$

$$I_B = j\frac{\sqrt{3}}{3}(a^2 I_1 - a I_2) = \frac{I_\beta}{3} \quad (60)$$

$$I_C = j\frac{\sqrt{3}}{3}(a I_1 - a^2 I_2) = \frac{I_\gamma}{3} \quad (61)$$

The connection of Figure 9 is adapted particularly to altering the zero-sequence component of current through the relay, which may be desirable because of the difference between Z_{0GH} and Z_{1GH} . Residual current may be introduced not only from the line being protected but also from a parallel or other adjacent line. Adjustable-ratio auxiliary transformers serve to introduce the desired proportions. Let r_1 and r_2 be the turn ratios of the auxiliary transformers, and $3I_0$ and $3I_0'$ be the currents in the primaries of these transformers, the primed symbol indicating the current from an adjacent line. Then the current feeding the neutral of the interconnected-star transformer is equal to $3(I_0 r_1 + I_0' r_2)$. The interconnected-star transformer divides this current into three equal portions, so that $(I_0 r_1 + I_0' r_2)$ is added to the line current in each relay. Let us designate these currents I_x , I_y , and I_z respectively

$$I_x = I_a + I_0 r_1 + I_0' r_2 \quad (62)$$

$$I_y = I_b + I_0 r_1 + I_0' r_2 \quad (63)$$

$$I_z = I_c + I_0 r_1 + I_0' r_2 \quad (64)$$

Equations 42, 43, and 44 for I_a , I_b , and I_c , equations 51, 52, and 53 for I_α , I_β , and I_γ , equations 59, 60, and 61 for I_A , I_B , and I_C , and equations 62, 63, and 64 for I_x , I_y , and I_z apply in general and for unbalanced faults involving ground. During line-to-line faults on an otherwise balanced system no ground current flows so that $I_0 = 0$, and equations 42, 43, and 44 become

$$I_a = I_1 + I_2 \quad (65)$$

$$I_b = a^2 I_1 + a I_2 \quad (66)$$

$$I_c = a I_1 + a^2 I_2 \quad (67)$$

Equations 62, 63, and 64 become

$$I_x = I_a = I_1 + I_2 \quad (68)$$

$$I_y = I_b = a^2 I_1 + a I_2 \quad (69)$$

$$I_z = I_c = a I_1 + a^2 I_2 \quad (70)$$

During balanced 3-phase faults, both I_2 and I_0 are zero, so that the foregoing become

$$I_a = I_1 \quad (71)$$

$$I_b = a^2 I_1 \quad (72)$$

$$I_c = a I_1 \quad (73)$$

$$I_\alpha = j\sqrt{3} I_1 = 3 I_A \quad (74)$$

$$I_\beta = j\sqrt{3} a^2 I_1 = 3 I_B \quad (75)$$

$$I_\gamma = j\sqrt{3} a I_1 = 3 I_C \quad (76)$$

$$I_A = j\frac{\sqrt{3}}{3} I_1 = \frac{1}{3} I_\alpha \quad (77)$$

$$I_B = j\frac{\sqrt{3}}{3} a^2 I_1 = \frac{1}{3} I_\beta \quad (78)$$

$$I_C = j\frac{\sqrt{3}}{3} a I_1 = \frac{1}{3} I_\gamma \quad (79)$$

$$I_x = I_1 \quad (80)$$

$$I_y = a^2 I_1 \quad (81)$$

$$I_z = a I_1 \quad (82)$$

The foregoing values are general and apply at any location. Hence, the expressions for the corresponding currents between particular points may be obtained by adding the proper subscripts, in accordance with Appendix I.

Appendix III. Line-to-Line Fault

Assume that a line-to-line fault has occurred at point F of one of the parallel lines of Figure 2, between conductors b and c , and that a fault resistance R_S is present between the two conductors. It will be convenient to consider that a resistance $R_S/2$ is connected to each line conductor at F and that at a point W the remaining terminals of the resistors on phases b and c are connected together, as shown in Figure 15a. This device has the advantage of making the system symmetrical up to the point W , at which point we have

$$E_{bW} = E_{cW} \quad (83)$$

$$I_{bFW} = -I_{cFW} \quad (84)$$

and

$$I_{aFW} = 0 \quad (85)$$

Resolving the currents into symmetrical components, we obtain

$$I_{1FW} = \frac{1}{3}(a - a^2)I_{bFW} = -I_{2FW} \quad (86)$$

$$I_{2FW} = \frac{1}{3}(a^2 - a)I_{bFW} = -I_{1FW} \quad (87)$$

$$I_{0FW} = 0 \quad (88)$$

As the system is assumed symmetrical at all other points, the fact that $I_{0FW} = 0$ requires that $E_0 = 0$ and $I_0 = 0$ over the entire system.

Resolving the voltages into symmetrical components gives

$$E_{1W} = \frac{1}{3}(E_{aW} - E_{bW}) = E_{2W} \quad (89)$$

$$E_{2W} = \frac{1}{3}(E_{aW} - E_{bW}) = E_{1W} \quad (90)$$

$$E_{0W} = \frac{1}{3}(E_{aW} + 2E_{bW}) = 0 \quad (91)$$

Hence

$$E_{aW} = -2E_{bW} = -2E_{cW} \quad (92)$$

and

$$E_{1W} = E_{2W} = -E_{bW} = -E_{cW} = E_{aW}/2 \quad (93)$$

These conditions will be satisfied if the networks are connected as shown in Figure 15b.²⁻⁴

As the system is symmetrical elsewhere, the sequence currents flow independently,¹⁻⁴ and the sequence potentials at point G may be obtained by adding to the sequence voltages at W the sequence voltage drops between W and G . The currents flowing from F to W are the sums of the currents flowing

from G to F and H to F . Making these substitutions, we obtain

$$E_G = I_{1GF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{1HF} \frac{R_S}{2} + E_{1W} \quad (94)$$

$$E_{2G} = I_{2G} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{2HF} \frac{R_S}{2} + E_{1W} \quad (95)$$

$$E_{0G} = 0 \quad (96)$$

The phase voltages at G become, from equations 39 to 41 and 48 to 50,

$$E_{aG} = I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} + 2E_{1W} \quad (97)$$

$$E_{bG} = I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} - E_{1W} \quad (98)$$

$$E_{cG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} - E_{1W} \quad (99)$$

$$E_{AG} = E_{cG} - E_{bG} = I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} \quad (100)$$

$$E_{BG} = E_{aG} - E_{cG} = I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} + 3E_{1W} \quad (101)$$

$$E_{CG} = E_{bG} - E_{aG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} - 3E_{1W} \quad (102)$$

For convenience in connection with star-current relaying connections, it is desirable to rewrite the foregoing in a different form. As $I_0 = 0$ everywhere during this type of fault, we may arbitrarily write $(I_{aGF} + I_{bGF} + I_{cGF})(mZ_{1GH} + R_S/2) + (I_{aHF} + I_{bHF} + I_{cHF})(R_S/2) = 0$.

Subtracting this equation from the foregoing gives

$$E_{AG} = -2I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{bHF} \frac{R_S}{2} - I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{aHF} \frac{R_S}{2} \quad (103)$$

$$E_{BG} = -2I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{cHF} \frac{R_S}{2} - I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{bHF} \frac{R_S}{2} + 3E_{1W} \quad (104)$$

$$E_{CG} = -2I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{aHF} \frac{R_S}{2} - I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{cHF} \frac{R_S}{2} - 3E_{1W} \quad (105)$$

As $I_{aGF} + I_{aHF} = I_{aFW} = 0$ equation 103 simplifies to

$$E_{AG} = -2I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{bHF} \frac{R_S}{2} - I_{aGF} mZ_{1GH} \quad (106)$$

Appendix IV. Double Line-to-Ground Fault

Assume that a double line-to-ground fault has occurred at point F of one of the parallel lines of Figure 2, between conductors b and c , that a fault resistance R_S is present between the two conductors, and that a fault resistance R_G occurs between some point of R_S and ground. Representation of the most general case requires that the point of connection of R_G may be at any point of R_S , but in order to obtain a simple analysis, it is necessary for R_G to be connected to the midpoint of R_S . In most practical cases, it is expected that either the resistance between phases will be small, or that the two parts will be nearly equal, so that the case assumed is sufficiently general for most purposes. It is convenient to consider again that a resistance of $R_S/2$ is connected to each line w re at the fault, and at a point W , the remaining terminals of the resistors on phases b and c are connected together and to one terminal of R_G , the remaining terminal of R_G being connected to ground. This is shown in Figure 16a.

Then at the point W , we have

$$a_{FW} = 0 \quad (107)$$

$$E_{dW} = E_{cW} = (I_{bFW} + I_{cFW})R_G \quad (108)$$

Resolving the currents into symmetrical components gives

$$I_{1FW} = \frac{1}{3}(aI_{bFW} + a^2I_{cFW}) \quad (109)$$

$$I_{2FW} = \frac{1}{3}(a^2I_{bFW} + aI_{cFW}) \quad (110)$$

$$I_{0FW} = \frac{1}{3}(I_{bFW} + I_{cFW}) \quad (111)$$

From which

$$I_{1FW} + I_{2FW} + I_{0FW} = 0 \quad (112)$$

and

$$I_{bFW} + I_{cFW} = 3I_{0FW} \quad (113)$$

Resolving the voltages into symmetrical components gives

$$E_{1W} = \frac{1}{3}(E_{aW} - 3I_{0FW}R_G) \quad (114)$$

$$E_{2W} = \frac{1}{3}(E_{aW} - 3I_{0FW}R_G) \quad (115)$$

$$E_{0W} = \frac{1}{3}(E_{aW} + 6I_{0FW}R_G) \quad (116)$$

from which

$$E_{1W} = E_{2W} = E_{0W} - 3I_{0FW}R_G \quad (117)$$

These conditions may be satisfied by connecting the networks as shown in Figure 16b.⁴ As all networks are symmetrical up to the point W , we may write the expressions for the voltages at the point G by adding

the proper sequence voltage drops. Remembering that the currents through the resistances $R_S/2$ are made up of the corresponding currents from G to F and H to F , we obtain

$$E_{1G} = I_{1GF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{1HF} \frac{R_S}{2} + E_{1W} \quad (118)$$

$$E_{2G} = I_{2GF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{2HF} \frac{R_S}{2} + E_{1W} \quad (119)$$

$$E_{0G} = I_{0GF} \left(mZ_{0GH} + \frac{R_S}{2} + 3R_G \right) + I_{0HF} \left(\frac{R_S}{2} + 3R_G \right) + I_{0GH} mZ_{0GH} + E_{1W} \quad (120)$$

The line-to-neutral and line-to-line voltages may be obtained by substituting in equations 39 to 41 and 48 to 50

$$E_{aG} = I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} + I_{0GF}(mZ_{0GH} - mZ_{1GH} + 3R_G) + 3I_{0HF}R_G + I_{0GH}mZ_{0GH} + 3E_{1W} \quad (121)$$

$$E_{bG} = I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} + I_{0GF}(mZ_{0GH} - mZ_{1GH} + 3R_G) + 3I_{0HF}R_G + I_{0GH}mZ_{0GH} \quad (122)$$

$$E_{cG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} + I_{0GF}(mZ_{0GH} - mZ_{1GH} + 3R_G) + 3I_{0HF}R_G + I_{0GH}mZ_{0GH} \quad (123)$$

$$E_{AG} = E_{cG} - E_{bG} = I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} \quad (124)$$

$$E_{BG} = E_{aG} - E_{cG} = I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} + 3E_{1W} \quad (125)$$

$$E_{CG} = E_{bG} - E_{aG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} - 3E_{1W} \quad (126)$$

For convenience in connection with star-current relaying connections, it is desirable to rewrite the foregoing in different form. As $I_{aGF} + I_{bGF} + I_{cGF} = 3I_{0GF}$ and $I_{aHF} + I_{bHF} + I_{cHF} = 3I_{0HF}$, we may write arbitrarily

$$(I_{aGF} + I_{bGF} + I_{cGF} - 3I_{0GF}) \times \left(mZ_{1GH} + \frac{R_S}{2} \right) + (I_{aHF} + I_{bHF} + I_{cHF} - 3I_{0HF}) \frac{R_S}{2} = 0$$

Subtracting this identity from the foregoing give:

$$\begin{aligned} E_{AG} = & -2I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{bHF} \frac{R_S}{2} + \\ & (3I_{0GF} - I_{aGF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (3I_{0HF} - I_{aHF}) \frac{R_S}{2} \quad (127) \end{aligned}$$

$$\begin{aligned} E_{BG} = & -2I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{cHF} \frac{R_S}{2} + \\ & (3I_{0GF} - I_{bGF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (3I_{0HF} - I_{bHF}) \frac{R_S}{2} + 3E_{1W} \quad (128) \end{aligned}$$

$$\begin{aligned} E_{CG} = & -2I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{aHF} \frac{R_S}{2} + \\ & (3I_{0GF} - I_{cGF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (3I_{0HF} - I_{cHF}) \frac{R_S}{2} - 3E_{1W} \quad (129) \end{aligned}$$

As $I_{aGF} = -I_{aHF}$, equation 127 may be simplified to

$$\begin{aligned} E_{AG} = & -2I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{bHF} \frac{R_S}{2} + \\ & 3I_{0GF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + 3I_{0HF} \frac{R_S}{2} - \\ & I_{aGF} mZ_{1GH} \quad (130) \end{aligned}$$

For studying ground relaying, it is desirable to have the line-to-neutral voltages expressed in terms of the symmetrical components of current. This may be done by substituting for the line currents in equations 121 to 123 inclusive, giving

$$\begin{aligned} E_{aG} = & (I_{1GF} + I_{2GF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (I_{1HF} + I_{2HF}) \frac{R_S}{2} + I_{0GF} \times \\ & \left(mZ_{0GH} + \frac{R_S}{2} + 3R_G \right) + \\ & I_{0HF} \left(\frac{R_S}{2} + 3R_G \right) + I_{0GH} mM_{0GH} + \\ & 3E_{1W} \quad (131) \end{aligned}$$

$$\begin{aligned} E_{bG} = & (\alpha^2 I_{1GF} + \alpha I_{2GF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (\alpha^2 I_{1HF} + \alpha I_{2HF}) \frac{R_S}{2} + I_{0GF} \times \\ & \left(mZ_{0GH} + \frac{R_S}{2} + 3R_G \right) + \\ & I_{0HF} \left(\frac{R_S}{2} + 3R_G \right) + I_{0GH} mM_{0GH} \quad (132) \end{aligned}$$

$$\begin{aligned} E_{cG} = & (\alpha I_{1GF} + \alpha^2 I_{2GF}) \left(mZ_{1GH} + \frac{R_S}{2} \right) + \\ & (\alpha I_{1HF} + \alpha^2 I_{2HF}) \frac{R_S}{2} + I_{0GF} \times \\ & \left(mZ_{0GH} + \frac{R_S}{2} + 3R_G \right) + \\ & I_{0HF} \left(\frac{R_S}{2} + 3R_G \right) + I_{0GH} mM_{0GH} \quad (133) \end{aligned}$$

Appendix V. Three-Phase Fault

Assume that a 3-phase fault has occurred at point F of one of the parallel lines of Figure 2, between all conductors and ground, and that a fault resistance $R_S/2$ is present between each conductor and ground. In the general case, the fault resistances will not be exactly equal, but the treatment of the general case is much more complicated, and the case of equal resistances is probably sufficiently general for our purposes. Making use of the previous assumption that a resistance $R_S/2$ is connected to each conductor at the fault, we may consider the remaining terminals connected together and to ground at point W , as shown in Figure 17a. Then we have

$$E_{aW} = E_{bW} = E_{cW} = 0 \quad (134)$$

$$I_{aFW} + I_{bFW} + I_{cFW} = 3I_{0FW} = 0 \quad (135)$$

From symmetry

$$I_{2FW} = 0 \quad (136)$$

These conditions are represented by the connection of Figure 17b. Hence,

$$E_{1G} = I_{1GF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{1HF} \frac{R_S}{2} \quad (137)$$

$$E_{2G} = 0 \quad (138)$$

$$E_{0G} = 0 \quad (139)$$

and

$$E_{aG} = I_{aGF} \left(mZ_{1HG} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} \quad (140)$$

$$E_{bG} = I_{bGF} \left(mZ_{1HG} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} \quad (141)$$

$$E_{cG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} \quad (142)$$

$$E_{AG} = I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{aHF} \frac{R_S}{2} \quad (143)$$

$$E_{BG} = I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{bHF} \frac{R_S}{2} \quad (144)$$

$$E_{CG} = I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) + I_{cHF} \frac{R_S}{2} \quad (145)$$

As the system is everywhere symmetrical, $I_{aGF} + I_{bGF} + I_{cGF} = 0$, hence $(I_{aGF} + I_{bGF} + I_{cGF})(mZ_{1GH} + R_S/2) = 0$. Likewise $(I_{aHF} + I_{bHF} + I_{cHF})(R_S/2) = 0$. Subtracting these expressions from the foregoing equations, we obtain the useful forms given below as

$$\begin{aligned} E_{AG} = & -2I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{bHF} \frac{R_S}{2} - \\ & I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{aHF} \frac{R_S}{2} \quad (146) \end{aligned}$$

$$\begin{aligned} E_{BG} = & -2I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{cHF} \frac{R_S}{2} - \\ & I_{bGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{bHF} \frac{R_S}{2} \quad (147) \end{aligned}$$

$$\begin{aligned} E_{CG} = & -2I_{aGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - 2I_{aHF} \frac{R_S}{2} - \\ & I_{cGF} \left(mZ_{1GH} + \frac{R_S}{2} \right) - I_{cHF} \frac{R_S}{2} \quad (148) \end{aligned}$$

Appendix VI. Single Line-to-Ground Fault

Assume that a single line-to-ground fault has occurred at a point F on one of the parallel lines of Figure 2, between conductor a and ground, and that a fault resistance R_G is present. It is convenient to assume that a resistance R_G is connected to each conductor at F and at a point W the other terminal of the resistance attached to conductor a is grounded, as shown in Figure 18a. Then the system is symmetrical up to the point W . At W we must have

$$\begin{aligned} I_{bFW} = I_{cFW} = 0 = I_{bGF} + I_{bHF} \\ = I_{cGF} + I_{cHF} \quad (149) \end{aligned}$$

and

$$E_{aW} = 0 \quad (150)$$

Resolving into components, we have

$$\begin{aligned} I_{1FW} = \frac{1}{3} I_{aFW} = I_{2FW} = I_{0FW} = I_{1GF} + I_{1HF} \\ = I_{2GF} + I_{2HF} = I_{0GF} + I_{0HF} \quad (151) \end{aligned}$$

$$E_{1W} = \frac{1}{3} (\alpha E_{bW} + \alpha^2 E_{cW}) \quad (152)$$

$$E_{2W} = \frac{1}{3} (\alpha^2 E_{bW} + \alpha E_{cW}) \quad (153)$$

$$E_{0W} = \frac{1}{3} (E_{bW} + E_{cW}) \quad (154)$$

so that

$$E_{1W} + E_{2W} + E_{0W} = 0 \quad (155)$$

These relations will be satisfied by connecting the sequence networks as shown in Figure 18b. It is immaterial, as far as the actual system is concerned, whether a resistance R_G is connected between each pair of networks as shown, or whether a resistance $3R_G$ is inserted between terminal F of the zero-sequence network and S of the positive-sequence network, as is more common.⁴ However, as some use is made in this analysis of the voltages at the terminals of the fictitious resistances connected to conductors b and c , the representation shown is preferred in this case.

Adding the voltage drops from G to W , we may obtain the voltages at G as

$$E_{1G} = I_{1GF} (mZ_{1GH} + R_G) + I_{1HF} R_G + E_{1W} \quad (156)$$

$$E_{2G} = I_{2GF} (mZ_{1GH} + R_G) + I_{2HF} R_G + E_{2W} \quad (157)$$

$$\begin{aligned} E_{0G} = & I_{0GF} (mZ_{0GH} + R_G) + I_{0HF} R_G + \\ & I_{0GH} mM_{0GH} + E_{0W} \quad (158) \end{aligned}$$

$$\begin{aligned} E_{aG} = & I_{aGF} (mZ_{1GH} + R_G) + I_{aHF} R_G + I_{0GF} m \\ & (Z_{0GH} - Z_{1GH}) + I_{0GH} mM_{0GH} \quad (159) \end{aligned}$$

$$\begin{aligned} E_{bG} = & I_{bGF} (mZ_{1GH} + R_G) + I_{bHF} R_G + \\ & I_{0GF} m (Z_{0GH} - Z_{1GH}) + I_{0GH} mM_{0GH} + \\ & j\sqrt{3} (\alpha E_{1W} - \alpha^2 E_{2W}) \quad (160) \end{aligned}$$

$$\begin{aligned} E_{cG} = & I_{cGF} (mZ_{1GH} + R_G) + I_{cHF} R_G + \\ & I_{0GF} (Z_{0GH} - Z_{1GH}) + I_{0GH} m \times \\ & M_{0GH} - j\sqrt{3} (\alpha^2 E_{1W} - \alpha E_{2W}) \quad (161) \end{aligned}$$

$$E_{AG} = I_{\alpha GF}(mZ_{1GH} + R_G) + I_{\alpha HF}R_G + j\sqrt{3}(E_{1W} - E_{2W}) \quad (162)$$

$$E_{BG} = I_{\beta GF}(mZ_{1GH} + R_G) + I_{\beta HF}R_G + j\sqrt{3}(a^2E_{1W} - aE_{2W}) \quad (163)$$

$$E_{CG} = I_{\gamma GF}(mZ_{1GH} + R_G) + I_{\gamma HF}R_G + j\sqrt{3}(aE_{1W} - a^2E_{2W}) \quad (164)$$

As $I_{bGF} = -I_{bHF}$ and $I_{cGF} = -I_{cHF}$, further simplification of equations 160 and 161 may be made, giving

$$E_{bG} = I_{bGF}mZ_{1GH} + I_{0GF}m(Z_{0GH} - Z_{1GH}) + I_{0GH}mM_{0GH} + j\sqrt{3}(aE_{1W} - a^2E_{2W}) \quad (165)$$

$$E_{cG} = I_{cGF}mZ_{1GH} + I_{0GF}m(Z_{0GH} - Z_{1GH}) + I_{0GH}mM_{0GH} - j\sqrt{3}(a^2E_{1W} - aE_{2W}) \quad (166)$$

For certain purposes it is desirable to have the foregoing voltages in terms of the symmetrical components of currents instead of the line currents. It also will be more convenient to combine all the resistance terms, making use of the relation that $I_{1GF} + I_{1HF} + I_{2GF} + I_{2HF} = 2(I_{0GF} + I_{0HF})$. Substituting and combining, we obtain

$$E_{aG} = (I_{1GF} + I_{2GF})mZ_{1GH} + I_{0GF}mZ_{0GH} + I_{0GH}mM_{0GH} + 3(I_{0GF} + I_{0HF})R_G \quad (167)$$

$$E_{bG} = (a^2I_{1GF} + aI_{2GF})mZ_{1GH} + I_{0GF}mZ_{0GH} + I_{0GH}mM_{0GH} + j\sqrt{3}(aE_{1W} - a^2E_{2W}) \quad (168)$$

$$E_{cG} = (aI_{1GF} + a^2I_{2GF})mZ_{1GH} + I_{0GF}mZ_{0GH} + I_{0GH}mM_{0GH} - j\sqrt{3}(a^2E_{1W} - aE_{2W}) \quad (169)$$

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Discussion

C. G. Dewey (General Electric Company, Philadelphia, Pa.): In connection with the discussion of fault resistance, it is worth noting that there is nearly the same disadvantage in having the second zone balance point to retract to a point within the first section, as for the first zone balance point to extend into the second section. The former also may cause loss of selectivity, which would mean losing power unnecessarily at some substation.

Even if it is granted that the balance point for no arc resistance can be set nearer the end of the section for the impedance relay than for the reactance, the protection is not necessarily better. This advantage is lost directly on arcing faults, as such faults cause an automatic retraction of reach in the impedance relay, particularly on short lines.

E. L. Harder (Westinghouse Electric Corporation, East Pittsburgh, Pa.): The method of symmetrical components now enters vitally into the design and application

of nearly every piece of equipment that goes into an electric power system. It is used in determining the short-circuit currents for which the generators and transformers must be braced. It is used in determining the interrupting duty of the circuit breakers and fuses. It enters into the application of lightning arresters and protector tubes, and into the insulation of transformers and other equipment. However, nowhere has the method of symmetrical components contributed quite so much as in the field of protective relaying. For in this field it has become the indispensable tool for visualizing, as well as computing, the currents and voltages during fault conditions, which are the fundamental basis of relay operation.

The introduction of the method of symmetrical components by the late Doctor Fortescue was followed by its expansion and development into a general industry tool by many of his associates, notably by Wagner and Evans, who developed and first presented in book form the working data and methods for treating machines, lines, and transformers. Wright, under their direction, and Kilgore determined

the sequence impedances of machines by test and calculation.

It perhaps is not surprising, therefore, that this paper, an excellent contribution of symmetrical components to the relaying field, has come from the hands of a man who was closely associated with Doctor Fortescue, Wagner, and Evans.

At the time of the writing of this paper the symmetrical component method was in quite common use for relay current calculations. However, distance relays, particularly of the high-speed type, were in their infancy, and little was known of the merits of different arrangements. Expression of the relay balance points in terms of the sequence impedances and currents of the system, provided, as the title implies, "A Fundamental Basis for Distance Relaying."

It showed immediately that the delta currents should be used to avoid a 15 per cent difference in the reach of the relay for 3-phase and line-to-line faults. It provided an accurate basis for appraising the relative merits of reactance and impedance relays. In this paper appear the fundamental formulas that show the two ways to make

ground distance relays measure correctly. Subsequently ground distance relays have been developed along both of these lines.

The work is so systematic, so thorough, and so fundamental that it stands today, as when written—the fundamental basis for distance relaying.

Edward W. Kimbark (Northwestern University, Evanston, Ill.): The new presentation and publication of an AIEE paper 16 years after it was first presented at an AIEE meeting and published in pamphlet form is, I believe, without precedent. The paper (31-61) first was presented during a period of depression when, for economy, many AIEE papers, including this one, were published in abstract in *ELECTRICAL ENGINEERING*. The unprecedented action now puts this paper on permanent, accessible record, where it deserves to be.

The passage of 16 years has but confirmed

earlier judgment that this paper by Lewis and Tippet is the classic paper in the field of power-system circuit theory as applied to predicting the behavior of protective relays. Nothing equalling it in this field had appeared before, and very little such has appeared since. Attempts to analyze the behavior of directional elements as Lewis and Tippet analyzed distance elements have produced results far less clear-cut than theirs. Only two other outstanding analyses in this field come to the mind of this discussor. They are the analyses of relay characteristics made by Neher¹ and Warrington² and the analysis of impedances seen by relays during swinging and out-of-step conditions made by Miss Clarke.³

Since the first publication of the paper by Lewis and Tippet, the following related developments have occurred:

1. The relay connections recommended in the paper have been used widely for phase relays and, to a lesser extent, for ground relays.

2. The analysis was extended by Calabrese⁴ to cover the case of relays on the side of a Y-delta-connected power transformer bank away from the fault.

3. An alternative method of obtaining the effect of delta currents, or differences of two line currents, was devised, in which the relay had two current windings each carrying one of the line currents.

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