

### Math 51100 Homework 3

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1. (4) Find the  $QR$  decomposition of the matrix  $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ .

2. (4) Show that if  $W$  is a subspace of  $\mathbb{R}^n$ , for any vector  $b \in \mathbb{R}^n$  the vector  $b - \text{proj}_W b$  is orthogonal to  $W$ .

3. (4) Let  $W = \text{span}\{ \langle 1, -1, 1 \rangle, \langle 0, 1, 2 \rangle \}$ . Let  $v = \langle 2, -3, 4 \rangle$ . Find an decomposition  $v = w_1 + w_2$ , where  $w_1 \in W$  and  $w_2 \in W^\perp$ .

4. (4) Fitting (in the least-square sense) the best parabola  $b = C + Dt + Et^2$  through the four data points  $(t, b) = (-1, 0), (0, 1), (1, 3), (2, K)$  is the same as solving what system of equations  $Ax = b$  by least-square? Set up (but do not solve) the matrix **equation** with which we can determine the least-square solution? Is there any value of  $K$  for which the system  $Ax = b$  has an exact solution? If your answer is “yes,” determine that value of  $K$ .

5. (5) Decide if the following statements are true or false. Justify your answer.

(a) If  $A$  is  $n \times n$  and invertible, then the projection of any vector in  $\mathbf{R}^n$  onto the column space of  $A$  will have zero error .

(b) Every projection matrix  $P$  is invertible.

(c) Any projection matrix  $P$  is idempotent (i.e.  $P^2 = P$ ), and so is  $I - P$  .

(d) If  $A$  is an  $n \times n$  matrix (not necessarily invertible), then the projection matrices associated with the row and column spaces are identical.

(e) The rank of  $AA^T$  is always equal to the rank of  $A^T$  .

6. (4) Find the determinants of

$$A = \begin{pmatrix} 2 & 0 & -2 & 1 \\ 0 & 3 & 3 & 1 \\ -1 & 4 & 0 & 2 \\ 3 & -2 & 4 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}.$$

7. (4) Let  $A$  be an  $n \times n$  matrix with  $\det A = 5$ . Find  $\det(2A)$ ,  $\det(-A^{-1})$ ,  $\det(A^T \cdot A)$ ,  $\det(A^5)$ .

8. (4) Find  $A^{-1}$ , where  $A = \begin{pmatrix} 2x+1 & x & 3x+2 \\ 0 & 4 & 0 \\ 4x-4 & 1 & 6x-5 \end{pmatrix}$ .

9. (4) Solve the system of equations by using Cramer's Rule:

$$\begin{cases} 2x - 2y + z = 7 \\ 5x + 2y - 3z = 0 \\ 4x + y + 2z = -1 \end{cases}$$

10. (4) Let  $\text{adj}(A)$  be the classical adjoint of the  $n \times n$  matrix  $A$ . Suppose  $\det A \neq 0$ . What is  $\det(\text{adj}(A))$ ? Justify.