

1. Consider the dispersive third order wave equation

$$u_t - cu_x + \nu u_{xxx} = 0, t > 0, -\infty < x < +\infty, c, \nu \text{ constants},$$

1a) Find solutions of the form

$$u(x, t; k) = \exp[i(kx - \omega(k)t)],$$

for any real k .

1b) Calculate the dispersion relation $\omega(k)$, the phase velocity $v_{ph}(k)$, and the group velocity $v_{gp}(k)$.

1c) Write down the general solution of this wave equation, as an integral over k .

1d) Write down the unique solution of this wave equation, as an integral over k , that satisfies the initial data $u(x, t=0) = u_{in}(x)$.

1e) From this Fourier representation of the solution of the initial value problem, calculate an approximation of $u(x, t)$ as $x \rightarrow \infty, t \rightarrow +\infty, x/t = v_{gr}$ held constant. (Note, the behavior will be different as $x \rightarrow \pm\infty$.)

1f) From the approximation of 1e, *describe in words* the qualitative behavior of the solution $u(x, t)$ as $x \rightarrow \infty, t \rightarrow +\infty, x/t = v_{gr}$.