

Combination of Functions.

1. a) $S.A = 4\pi r^2$

$$S.A = 12.568 r^2$$

b) $V = \frac{4}{3}\pi r^3$

$$3V = 4\pi r^3$$

$$4\pi r^3 = 3V$$

$$r^3 = \frac{3V}{4\pi}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

or

$$r = 2.325 \sqrt[3]{3V}$$

c) $S.A(r(V))$

$$12.568 r^2 \left(2.325 \sqrt[3]{3V} \right)^2$$

$$12.568 r^2 \left(9.7402 r^3 \sqrt[3]{V} \right)$$

$$122.4148 r^5 \sqrt[3]{V}$$

d) $V = \frac{4}{3}\pi r^3$

$$0.75 = \frac{4}{3} \times 3.142 \times r^3$$

$$r^3 = \frac{0.75 \times 3}{4 \times 3.142}$$

$$r^3 = 0.17903$$

$$r = 0.5636$$

$$S.A = 4\pi r^2$$

$$= 4 \times 3.142 \times 0.5636^2$$

$$= 1.268 m^2$$

2. a) $f \circ g(x)$

$$f(x) = x+2$$

$$g(x) = x^2 - 4$$

$$f \circ g(x) = x^2 - 4 + 2$$

$$f \circ g(x) = x^2 - 2$$

b) $g \circ f(x)$

$$(x+2)^2 - 4$$

$$(x+2)(x+2) - 4$$

$$x^2 + 4x + 4 - 4$$

$$x^2 + 4x$$

$$g \circ f(x) = x^2 + 4x$$

or

$$x(x+4)$$

c) $(h+g)(-1)$

$$h(x) = \frac{1}{\sqrt{x+1}} \quad g(x) = x^2 - 4$$

$$(h+g)(-1)$$

$$\left(\frac{1}{\sqrt{x+1}} + x^2 - 4 \right) (-1)$$

$$= \frac{1}{\sqrt{(-1)+1}} + (-1)^2 - 4$$

$$\sqrt{(-1)+1}$$

$$= \frac{1}{0} + 1 - 4$$

$$= \alpha$$

Exponential and Logarithmic Functions

$$5. a) 5^{x+1} = 6^{x+1}$$
$$\log 5^{x+1} = \log 6^{x+1}$$

$$(x+1) \log 5 = (x+1) \log 6$$

$$\frac{x+1}{x+1} = \frac{\log 6}{\log 5}$$

$$\frac{x+1}{x+1} = 1.113$$

$$1.113x + 1.113 = x + 1$$

$$1.113x - x = 1 - 1.113$$

$$0.113x = -0.113$$

$$x = \frac{-0.113}{0.113}$$

$$x = -1$$

$$b. 3^{5x} \cdot 9^{x^2} = 27$$

$$3^{5x} \times 3^{2x^2} = 3^3$$

$$5x + 2x^2 = 3$$

$$2x^2 + 5x - 3 = 0$$

$$a+b=5 \quad (6, -1)$$

$$a \times b = -6$$

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(2x-1)(x+3) = 0$$

$$x = -3 \text{ or } \frac{1}{2}$$

$$6. a) t = \log \left(\frac{T-22}{25} \right) \div \log 0.75$$

$$T = 35^\circ\text{C}$$

$$t = \log \left(\frac{35-22}{25} \right) \div \log 0.75$$

$$t = \log 0.52 \div \log 0.75$$

$$t = 2.273$$

$$\approx 2 \text{ minutes}$$

$$b. \text{Initial temperature } t = 0$$

$$0 = \log \left(\frac{T-22}{25} \right) \div \log 0.75$$

$$\log 1 = \log \left(\frac{T-22}{16.5} \right)$$

$$\frac{T-22}{16.5} = 1$$

$$T-22 = 16.5$$

$$T = 16.5 + 22$$

$$T = 38.5^\circ\text{C}$$

Exponential and Logarithmic Functions.

1. a) $\log_3 \frac{1}{9}$

$$\log_3 9^{-1} = \log_3 3^{-2}$$

$$= -2 \log_3 3$$

$$\log_3 3 = 1$$

$$= -2 \times 1$$

$$= -2$$

b) $\log_5 100 - \log_5 4$

$$\log_5 \left(\frac{100}{4} \right) = \log_5 25$$

$$\log_5 5^2 = 2 \log_5 5$$

$$= 2$$

c) $\log 2 + \log 5 - \log 25$

$$\log(2 \times 5) - \log_5 5^2$$

$$\log 10 - 2 \log_5 5$$

$$\underline{\quad \quad}$$

$$1 - 2 = -1$$

3. $\log a + \log b = \log ab$

$$a \log b = \log(b^a)$$

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

4 a) $\log_4(x+2) + \log_4(x-1) = 1$

$$\log_4(x+2) + \log_4(x-1) = \log_4 4$$

$$\log_4(x+2)(x-1) = \log_4 4$$

$$(x+2)(x-1) = 4$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 2 - 4 = 0$$

$$x^2 + x - 6 = 0$$

$$a+b=1 \quad (-2, 3)$$

$$a \times b = -6$$

$$x^2 - 2x + 3x - 6 = 0$$

$$x(x-2) + 2(x-2) = 0$$

$$(x+3)(x-2) = 0$$

$$x = 2 \text{ or } -3$$

2. $\log_4 x^2 + 3 \log_3 \frac{1}{27} y - \log_4 x$

$$\log_4 \left(x^2 \times \frac{1}{27} y^3 \times \frac{1}{x} \right)$$

$$\log_4 \left(\frac{x^2 y^3}{27} \right)$$

$$= \log_4 \left(\frac{x^2 y^3}{27} \right)$$

Exponential and Logarithmic Functions

b) $\log_3(8x-2) + \log_3(x-1) = 2$

$$\log_3(8x-2) + \log_3(x-1) = \log_3 9$$

$$\log_3(8x-2)(x-1) = \log_3 9$$

$$(8x-2)(x-1) = 9$$

$$8x^2 - 10x + 2 = 9$$

$$8x^2 - 10x + 2 - 9 = 0$$

$$8x^2 - 10x - 7 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 8 \times -7}}{2 \times 8}$$

$$x = \frac{10 \pm 18}{16}$$

$$x = \frac{10+18}{16} \text{ or } \frac{10-18}{16}$$

$$= 1.75 \text{ or } -0.5$$

Trigonometric functions.

1. $\cos^4 x - \sin^4 x = \cos 2x$

$$x^2 - y^2 = (x+y)(x-y)$$

$$\cos^4 x - \sin^4 x =$$

$$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\text{A } \cos^4 x - \sin^4 x = \cos 2x$$

2. a) $\frac{2}{\sin x} + 10 = 6$

$$\frac{2}{\sin x} = -4$$

$$\sin x = \frac{2}{-4}$$

$$\sin x = -0.5$$

$$x = \sin^{-1} 0.5$$

$$x = \frac{\pi}{6}$$

where x is negative.

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$= 210^\circ, 330^\circ$$

b) $3 + 10 \sec x - 1 = -18$

$$10 \sec x = -18 - 3 + 1$$

$$10 \sec x = -20$$

$$\sec x = -2$$

$$x = \sec^{-1} -2$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{1}{\cos x} = -2$$

$$\cos x = -0.5$$

$$x = \cos^{-1} 0.5$$

$$x = 60^\circ = \frac{\pi}{3}$$

where x is negative

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$= 120^\circ, 240^\circ$$

3 a) $(2 \sin x - 1)(\cos x) = 0$

$$2 \sin x - 1 = 0 \quad \cos x = 0$$

$$2 \sin x = 1 \quad x = \cos^{-1} 0$$

$$\sin x = 0.5 \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \sin^{-1} 0.5 \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad = 90^\circ, 270^\circ$$

$$= 30^\circ, 150^\circ$$

$$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$= 30^\circ, 90^\circ, 150^\circ, 270^\circ$$

Trigonometric Functions.

b) $\cos^2 x = 2 + \cos x$

$$\cos^2 x - \cos x - 2 = 0$$

$$a+b = -1$$

$$a \times b = -2$$

$$\cos^2 x - 2(\cos x) + (\cos x) - 2 = 0$$

$$\cos x (\cos x - 2) + 1(\cos x - 2) = 0$$

$$(\cos x + 1)(\cos x - 2) = 0$$

$$\cos x + 1 = 0$$

$$\cos x - 2 = 0$$

$$\cos x = -1$$

$$\cos x = 2$$

$$x = \cos^{-1}(-1)$$

$$x = \cos^{-1}(2)$$

$$x = 180^\circ = \pi$$

$$x = 0$$

$$\therefore x = 0^\circ, 180^\circ$$

$$= \pi$$

A. $d(t) = 4 + 2 \sin\left(\frac{\pi}{6}t\right)$

$$d = 3$$

$$3(t) = 4 + 2 \sin \frac{\pi}{6} t$$

$$3(t) - 2 \sin \frac{\pi}{6} t = 4$$

$$3t - 2 \times 0.5t = 4$$

$$3t - t = 4$$

$$2t = 4$$

$$t = 2 \text{ hours}$$

5. $2 \sin x \sec x - 2\sqrt{3} \sin x = 0$

$$\sin x \sec x - \sqrt{3} \sin x = 0$$

$$\sin x \sec x = \sqrt{3} \sin x$$

$$\sec x = \sqrt{3}$$

$$\frac{1}{\cos x} = \sqrt{3}$$

$$\cos x$$

$$\cos x = \frac{1}{\sqrt{3}}$$

$$x = \cos^{-1} \frac{1}{\sqrt{3}}$$

$$x = \cancel{0.499} 54.7353^\circ$$

$$x = 54.7353^\circ, 305.2647^\circ$$

$$x = 54.74^\circ, 305.26^\circ$$

Combination of Functions.

3. $C(x) = 5x + 18$

$R(x) = 2x^2$

$2x^2 = 5x + 18$

$2x^2 - 5x - 18 = 0$

$a+b = -5 \quad -9, 4$

$a \times b = -36$

$2x^2 - 9x + 4x + 18 = 0$

$x(2x - 9) + 2(2x - 9) = 0$

$(x+2)(2x-9) = 0$

$x+2 = 0 \quad 2x-9 = 0$

$x = -2 \quad 2x = 9$

$x = 4.5$

$x = -2 \text{ or } 4.5$

$\therefore$ Number of product = 4.5.

4a) $f(x) = 4 \tan 5x$

$g(x) = 3 \cos 5x$

$(f \times g)(x)$

$= 4 \tan 5(3 \cos 5x)$

$(f \times g)(x) = 4 \tan(5 \cdot 3 \cos(5x))$

b) $f(x) = \sqrt{5x^2}$ and $g(x) = 5\sqrt{5x^2}$

$(f \times g)(x)$

$= \sqrt{5(5\sqrt{5x^2})^2}$

$= \sqrt{5(125x^2)}$

$= \sqrt{625x^2}$

$(f \times g)(x) = 25\sqrt{x^2}$