

b. Perpendicular to ~~to~~

$$(2, -2, 3)$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos 90^\circ$$

$$\text{but } \cos 90^\circ = 0$$

$$\text{So } \vec{a} \cdot \vec{b} = 0$$

$$(2, -2, 3) \cdot (a_1, a_2, a_3) = 0$$

$$a_2 = -1$$

$$a_3 = 2$$

Then

$$2a_1 - 2(-1) + 3(2) = 0$$

$$2a_1 + 2 + 6 = 0$$

$$2a_1 = -8$$

$$a_1 = -4$$

$$= (-4, -1, 2)$$

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \\ &= \frac{20}{\sqrt{19} \cdot \sqrt{14}} \\ &= \frac{20}{32.83}\end{aligned}$$

$$\begin{aligned}\cos \theta &= 0.6091 \\ &= \underline{\underline{52.48^\circ}}\end{aligned}$$

Question 3

$$A(1, 5, -2)$$

$$B(0, 2, -4)$$

(a) The vector \vec{BA}

$$(1-0)$$

$$(1-0, 5-2, -2+4)$$

$$= (1, 3, 2)$$

$$(b) \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{AB}|^2}{2|\vec{a}||\vec{b}|}$$

$$\sqrt{1^2 + 3^2 + 2^2}$$

$$= 1 + 9 + 4$$

$$= \sqrt{14}$$

Question 4

a (parallel to $(1, 1, 1)$)

$$(2, -2, 3)$$

two vectors are parallel iff

$$u_1, u_2, u_3 = c(u_1, u_2, u_3) \text{ where } c \text{ is a constant.}$$

$$\text{So } (2, -2, 3) = 2(2, -2, 3)$$

$$= (4, -4, 6)$$

For parallel

$$(2, 12) \quad (6, k)$$

$$\vec{u} \begin{pmatrix} 2 \\ -12 \end{pmatrix} \quad \begin{pmatrix} 6 \\ k \end{pmatrix}$$

$$2n \in 6$$

$$n = 3$$

$$-12n = k$$

$$-12(3) = -36$$

$$k = -36$$

Question 2

$$\vec{u} = (3, 8, 2) \quad \vec{v} = (-2, 3, 1)$$

(a) $\vec{u} \cdot \vec{v}$

$$\begin{aligned} & u_1 v_1 + u_2 v_2 + u_3 v_3 \\ & 3 \times (-2) + 8 \times 3 + 2 \times 1 \\ & -6 + 24 + 2 \\ & = 20 \end{aligned}$$

(b) $\vec{u} \times \vec{v}$

$$c_x = u_2 v_3 - u_3 v_2 = 8(3) - (2)(3) = 2$$

$$c_y = u_3 v_1 - u_1 v_3 = (2)(-2) - (3)(1) = -7$$

$$c_z = u_1 v_2 - u_2 v_1 = (3)(3) - (8)(-2) = 29$$

$$\vec{a} \times \vec{b} = (2, -7, 29)$$

(c)

$$|\vec{u}| = \sqrt{(2^2 + 8^2 + 2^2)} = \sqrt{4 + 64 + 4} = \sqrt{72}$$

$$|\vec{v}| = \sqrt{((-2)^2 + 3^2 + 1^2)} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

Question 9

$$\vec{a} = 2, -2, 4 \quad \vec{b} = (0, 2, 3)$$

$$\vec{a} \cdot \vec{b}$$

$$a \cdot b = a_x \times b_x + a_y \times b_y + a_z \times b_z$$

$$= (2 \times 0) + (-2 \times 2) + (4 \times 3)$$

$$0 - 4 + 12$$

$$= 8$$

$$\vec{a} \times \vec{b}$$

a

b

$$(2, -2, 4) \quad (0, 2, 3)$$

$$c_x = a_y b_z - a_z b_y = (-2 \times 3) - (4 \times 2) = -6 - 8 = -14$$

$$c_y = a_z b_x - a_x b_z = (4 \times 0) - (2 \times 3) = 0 - 6 = -6$$

$$c_z = a_x b_y - a_y b_x = (2 \times 2) - (-2 \times 0) = 4 - 0 = 4$$

$$\vec{a} \times \vec{b} = (-14, -6, 4)$$

Question 1

$$u = (2i - 12k) \quad v = (6i + k)$$

u and v are perpendicular to v

$$\vec{u} \cdot \vec{v} = 0$$

$$(2, -12) \cdot (6, k) = 0$$

$$12 - 12k = 0$$

$$-12k = -12$$

$$k = 1$$