

Lab: Buoyancy

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Introduction: Buoyancy is the reason why objects float. In this experiment we'll use the theoretical basis for buoyancy to predict the mass of a glass, and then if you have a scale you can compare with a measured mass.

Theory: The reason why objects float is because a force acts upwards on them, counteracting gravity. This force is called the buoyant force. It exists whenever an object becomes at least partially submerged in a fluid. The formula for the buoyant force is typically written $F_B = \rho V g$, where F_B is the buoyant force, ρ is the commonly used symbol for density (mass divided by volume,) V stands for volume, and g is the acceleration due to gravity. The formula as written is unclear - which density and volume should be used? I prefer to write the formula with some more detail:

$$F_B = \rho_f V_{\text{sub}} g \quad (1)$$

Now, ρ_f is the density of the fluid that the object is being submerged into. V_{sub} is the submerged volume of the object (if the object is totally submerged, then V_{sub} is the entire volume of the object, if it's only half-submerged then V_{sub} is half the volume of the object.) To see this in action, note that for an object to float, the totally submerged buoyant force must exceed the gravitational force. We can write the gravitational force on the object as $mg = \rho_o V g$ where ρ_o is the density of the object and V is the total volume of the object (this is just using the definition, $\rho = m/V$.) Writing $F_B > F_G$ (with $V_{\text{sub}} = V$ since it's totally submerged) then gives the simple relation $\rho_f > \rho_o$. An object will float in a fluid if the fluid density is greater than the density of the object.

Procedure: Find a glass or cup, preferably one with a round shape like the diagram.

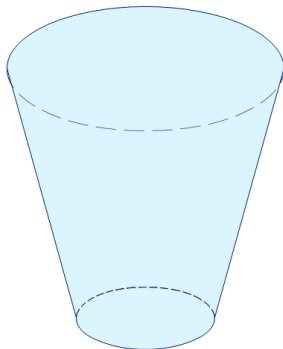
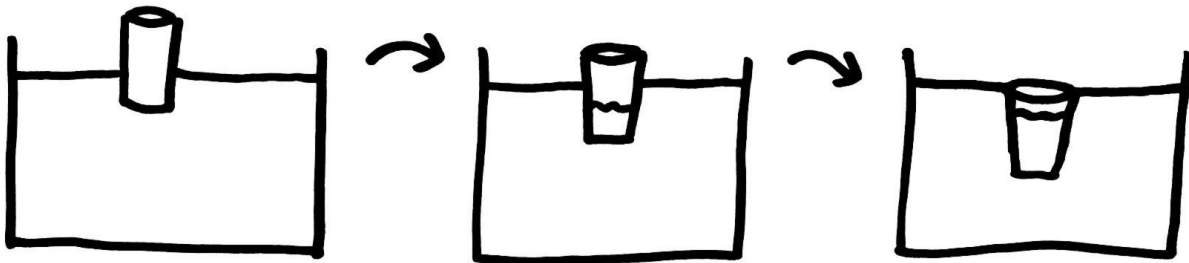


Fig. 1. Image courtesy of conceptdraw.com/a1942c3/preview

Then, fill up either a big pot or a sink with water, high enough to cover the entire glass. Once the pot is full, place the glass in the water with its opening facing upwards. The base of the glass should drop a little into the water, and then float. Then, begin slowly pouring water into the glass until the rim of the glass is at the same level as the water in the pot. You should notice that the water level in the glass is slightly below the level of the water in the pot.



Now you have found the point at which the glass is *barely* floating while (close to) totally submerged. There are three forces acting on the glass, its own mass causes gravity to pull it down, the buoyant force from the water in the pot pushes it up, and the water in the glass itself pushes the glass down. Since it's not accelerating, the net force is zero. Therefore the total upward force equals the total downward force:

$$\rho_w V g = m g + m_w g. \quad (2)$$

Here ρ_w is the density of water, V is the total volume of the glass (including the inside), m is the mass of the glass itself, and m_w is the mass of the water inside the glass. We can write the mass of the water inside the glass as the density of water times the volume of water inside the glass, $m_w = \rho_w V_w$. Then, rearranging the equation to solve for the mass of the glass,

$$m = \rho_w (V - V_w). \quad (3)$$

Now, to calculate the volume of the glass, we'll approximate it to have exactly the shape suggested in Fig.1. Measure the radius at the bottom of the glass, call that r_1 (pour the water into a separate glass for a moment if you need to turn the glass upside down to measure the radius.) Measure also the radius at the top of the glass and call that r_2 . Then, measure the height of the glass, h . To get the volume out of these three measurements requires either calculus or some clever geometry, we'll simply state and use the result:

$$V = \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2). \quad (4)$$

Once you have this volume, we then just need the volume of water inside the glass. If you have a measuring cup or other kitchen tool to measure volume, you can use that (just make sure to convert back to metric units!) If not, simply use the same volume formula as above but using h as the height of the water in the glass, instead of the height of the glass itself, and use r_2 as the radius at the level

of the water. Note: if your glass has a thick bottom, you may want to measure the height of the water from the point where the water begins, not the bottom of the glass (and similarly for the radius r_1 .) So, now that you have the two volumes, we can return to Eq. 3 and use the known density of water $\rho_w = 1000 \text{ kg/m}^3$ to calculate the mass of the glass, m . Since this value of m came from the theoretical formula for the buoyant force, this is our theoretical mass m_{theory} .

To measure m_{meas} is a much simpler task. If you have a kitchen scale, simply place it on the scale and write down the mass. If you don't have a scale, you may skip this part, although it will be more interesting if you can at least get an idea of the mass. One method you can use is to look in your pantry or fridge for some food items that have a listed weight (probably in ounces - you'll need to do some converting.) You can set up a rudimentary balance scale by placing items on opposite sides of a long object and then balancing that object at its center (similar to the torque experiment!) This method of measurement will likely have large error, but you should mention in your report whether or not you think your theoretical measurement is at least reasonable.

Analysis: If you are able to get a value for m_{meas} , then you can use the usual % error formula:

$$\% \text{ error} = \frac{|m_{\text{meas}} - m_{\text{theory}}|}{m_{\text{theory}}} \times 100\%. \quad (5)$$

Conclusion: What are some sources of error in this experiment? The density of wood is less than the density of water, so it immediately makes sense that wooden boats float. But, nowadays people make boats out of steel. How is it possible that steel boats float even though steel has a density roughly 8 times higher than water?