

Lab: How Big is the Moon?

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Introduction: In this experiment, we will try to find the size of the Moon. Newton's laws can help us to predict the distance between the Earth and the Moon, but not the diameter of the moon itself.

Theory: Newton's law of gravitation says that the force of gravity between two massive objects is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

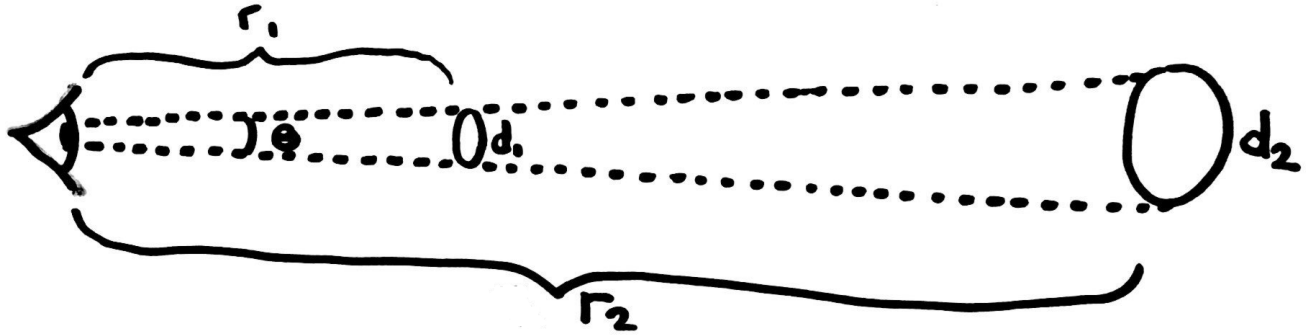
$$F_G = G \frac{m_e m_m}{r^2} \quad (1)$$

G is a constant called Newton's gravitational constant. Its value is $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$. Here m_e is the mass of the Earth, $m_e = 6 \times 10^{24} \text{ kg}$, m_m is the mass of the Moon, and r is the distance between the Earth and the Moon. For an object moving in a circle of radius r , its acceleration is v^2/r where v is its speed. Assume that the Moon orbits the Earth in a circle, and use Newton's law of gravity to show that

$$r^3 = \frac{G m_e}{4\pi^2} T^2. \quad (2)$$

Here T is the time period for a full lunar revolution. You'll want to use the fact that speed is distance divided by time, so you can write v as the circumference of the circle divided by the time period T . Once you have Eq. 2, note that the Moon is observed to orbit the Earth in $T = 27$ days. Use this fact (converted to seconds) to derive the distance between them. Compare your result with the accepted value of $r = 3.844 \times 10^8 \text{ m}$.

Procedure: Now that we know the distance to the Moon, let's try to figure out its diameter. Find a small coin, button, or other round object and go outside in the evening (or look out the window if you have a good view!) Find the Moon in the sky, and hold up the button. Close one eye and try to move the button so that it perfectly eclipses the Moon. In other words, find the furthest distance from your eye where you can hold the button and still completely block the Moon. If your object still appears larger than the moon even when your arm is fully extended, you will need to find a smaller object. Once you have found the right distance to hold the button, measure the distance from your eye to the button. You may want to use a piece of string for this, so you can then lay down the string afterwards next to a ruler. Lastly, you'll also want to measure the diameter of the button itself.



This diagram is *not* to scale! $r_1 \ll r_2$

Let's call the distance from your eye to the button r_1 , and the diameter of the button d_1 . Let's also call the distance from your eye to the moon r_2 , and the diameter of the moon d_2 . Since the angle θ is the same for both of these isosceles triangles, they are what we call *similar* triangles. That means the ratio of any two of their sides must be the same:

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} \quad (3)$$

We're interested in the Moon diameter d_2 , so we can rearrange the equation, solving for it. Before we do that however, note that you have measured both r_1 and d_1 . r_2 is the distance from your eye to the moon, which we have *not* yet found. In the theory section, you found the distance between the Earth and the moon, but gravity acts on the center of mass of objects, and your eye is located not at the center of the Earth, but at the surface. So we have $r_2 = r - r_e$ where r is your distance calculated in the theory section, and r_e is the radius of the Earth. So, our equation becomes

$$d_2 = (r - r_e) \frac{d_1}{r_1}. \quad (4)$$

Use the fact that $r_e = 6.371 \times 10^6 m$ to now calculate d_2 , your measured diameter of the Moon.

Analysis: Compare your measured Moon diameter with the known value of $d_{\text{known}} = 3.47 \times 10^6 m$ using the usual % error formula:

$$\% \text{ error} = \frac{|d_2 - d_{\text{known}}|}{d_{\text{known}}} \times 100\%. \quad (5)$$

Conclusion: What are the sources of error for this experiment? Suppose an astronaut has returned with Moon rocks, and you find the average density of these rocks is $3.34 \times 10^3 kg/m^3$. Using what we now know about the diameter of the Moon, estimate the Moon's total mass.