

1. (a) Negation.

$$\begin{aligned} & \sim((n < 1) \vee (n > 11)) \\ &= \sim(n < 1) \wedge \sim(n > 11) \end{aligned}$$

(b) n is not less than 1 and n is not greater than 11

(c) The logical value of P is sometimes true.
When $n = 4$, or 5 , the statement is false

(d) The logical value of P' is also sometimes true.
When n is outside the range, the statement is false

2 (a) $(P \wedge Q) \Rightarrow R$

(i) If I work hard and pass my finals, I will go to college

(ii) P = Work hard.
 Q = Pass my finals
 R = Go to college.

(b) $P \Leftrightarrow (Q' \vee S)$

(i) It is dry outside if and only if it is not raining or there is sunshine

(ii) P = It is dry outside
 Q' = It is not raining
 S = There is sunshine

3. P
 Q

3	P	Q	P'	Q'	$P \wedge Q$	$P' \wedge Q'$	$(P \wedge Q) \vee (P' \wedge Q')$
	T	T	F	F	T	F	T
	T	F	F	T	F	F	F
	F	T	T	F	F	F	F
	F	F	T	T	F	T	T

$P \Leftrightarrow Q$	$(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (P' \wedge Q')$
T	T
F	T
F	T
T	T

The last column has only T's (always true). Therefore, this formula is a tautology.

4. $((P \wedge (P \Rightarrow Q)) \Rightarrow Q) \equiv T$

$$\sim((P \wedge (\sim P \vee Q)) \vee Q)$$

Implication law, twice

$$\sim((P \wedge \sim P) \vee (P \wedge Q)) \vee Q$$

Distributive law

$$\sim(F \vee (P \wedge Q)) \vee Q$$

Negation law

$$\sim((F \vee P) \wedge (F \vee Q)) \vee Q$$

Distributive law

$$\sim((P \vee F) \wedge (Q \vee F)) \vee Q$$

Commutative law

$$\sim(P \wedge Q) \vee Q$$

Identity law

$$(\sim P \vee \sim Q) \vee Q$$

DeMorgan's law

$$\sim P \vee (\sim Q \vee Q)$$

Associative law

$$\sim P \vee T$$

Negation law

$$T$$

Identity law

5.

$P \vee Q$
 $P' \vee R$
 $R' \vee S$
 $\therefore Q \vee S$

P	Q	R	S	$P \vee Q$	P'	$P' \vee R$	R'	$R' \vee S$	$Q \vee S$	
T	T	T	T	<u>T</u>	F	<u>T</u>	F	<u>T</u>	T	Critical row
T	T	T	F	T	F	T	F	F	T	
T	T	F	T	T	F	F	T	T	T	
T	T	F	F	T	F	F	T	T	T	
T	F	T	T	<u>T</u>	F	<u>T</u>	F	<u>T</u>	T	Critical row
T	F	T	F	T	F	T	F	F	F	
T	F	F	T	T	F	F	T	T	T	
T	F	F	F	T	F	F	T	T	F	
F	T	T	T	<u>T</u>	T	<u>T</u>	F	<u>T</u>	T	Critical row
F	T	T	F	<u>T</u>	T	<u>T</u>	F	<u>T</u>	T	Critical row
F	T	F	T	<u>T</u>	T	<u>T</u>	T	<u>T</u>	T	Critical row
F	T	F	F	<u>T</u>	T	<u>T</u>	T	<u>T</u>	T	Critical row
F	F	T	T	F	T	T	F	T	T	
F	F	T	F	F	T	T	F	F	F	
F	F	F	T	F	T	T	T	T	T	
F	F	F	F	F	T	T	T	T	F	

All the critical rows have a true conclusion. So, the argument is valid.

6. $\sum_{k=1}^n (2k-1) = n^2$

$$S_n = 1 + 3 + 5 + 7 + \dots + (2n-1) = n^2$$

If $n=1$ $S_1 = 1 = 1^2$ This is true

Assume $n=k$

$$S_k = 1 + 3 + 5 + 7 \dots + (2k-1) = k^2 \text{ This is true}$$

If $n=k+1$, show that S_{k+1} is true

$$\begin{aligned} S_{k+1} &= 1 + 3 + 5 + 7 \dots + (2k-1) + (2(k+1)-1) = (k+1)^2 \\ &= 1 + 3 + 5 + 7 \dots + (2k-1) + (2k+1) = (k+1)^2 \end{aligned}$$

Add $(2k+1)$ on both sides

$$1 + 3 + 5 + 7 \dots + (2k-1) = (k+1)^2 + (2k+1)$$

$$1 + 3 + 5 + 7 \dots + (2k-1) = k^2 +$$

$$= \underbrace{1 + 3 + 5 + 7 \dots + (2k-1)}_{k^2} + (2k+1) = (k+1)^2$$

$$= k^2 + 2k + 1 = (k+1)^2$$

This is true

Since the statement is true for $n=1$ and $n=k$, it implied that $n=k+1$ is also true