

Langara College
Final Examination
Mathematics 1170
All sections

Duration: 1h 40 minutes to write + 15 minutes to scan/submit

Submission deadline: 5:55 PM

You should stop writing at 5:40 PM and start scanning! Late submission will receive 1% penalty per minute!

First Name _____ Last Name _____

Student Number _____

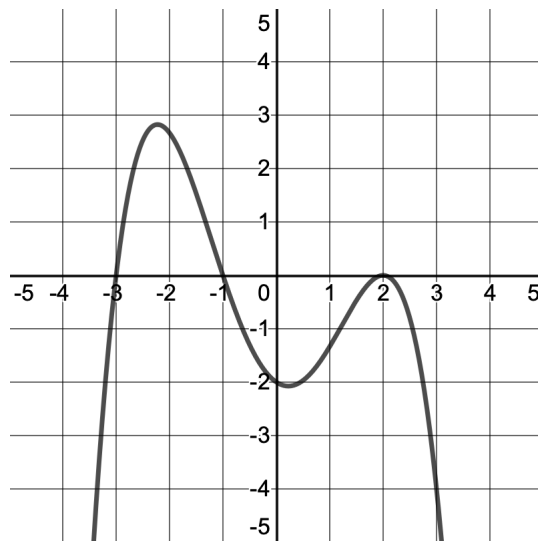
INSTRUCTIONS

- This is a **closed book** examination, and therefore, you **may not** use your lecture notes or the textbooks. You **may not** get help from anyone else or the Internet. You are permitted to use a non-graphing calculator, but no other electronic devices are allowed. Any evidence that you did not independently come up with the submitted solutions will result in appropriate disciplinary action.
- You can write your solutions on a print-out of the exam or write them directly on clean sheets of paper. You must ensure that your name and student number are on your submitted solutions. You must scan all the pages, combine them into a single pdf file, and **submit via Brightspace no later than 5:55 pm** on the exam date. *Submission via email will NOT be accepted for marking, and late submission will be subjected to deduction.*
- Your answers will be graded for clarity of exposition as well as correctness of result. For full marks, all appropriate work must be shown. Exact answers must be given when they can be found.
- Marks for each question are shown in square brackets []. There are 100 marks in total. Good luck!

1		5
2		5
3		15
4		5
5		10
6		10
7		7
8		4
9		5
10		15
11		10
12		9
Total		100

1. [5 marks] If $f(x) = \sqrt{7-3x}$, evaluate and simplify as much as possible the expression $\frac{f(1+h) - f(1)}{h}$ where $h \neq 0$.

2. [5 marks] Find an expression for the polynomial $f(x)$ of degree 4 whose graph is given below:



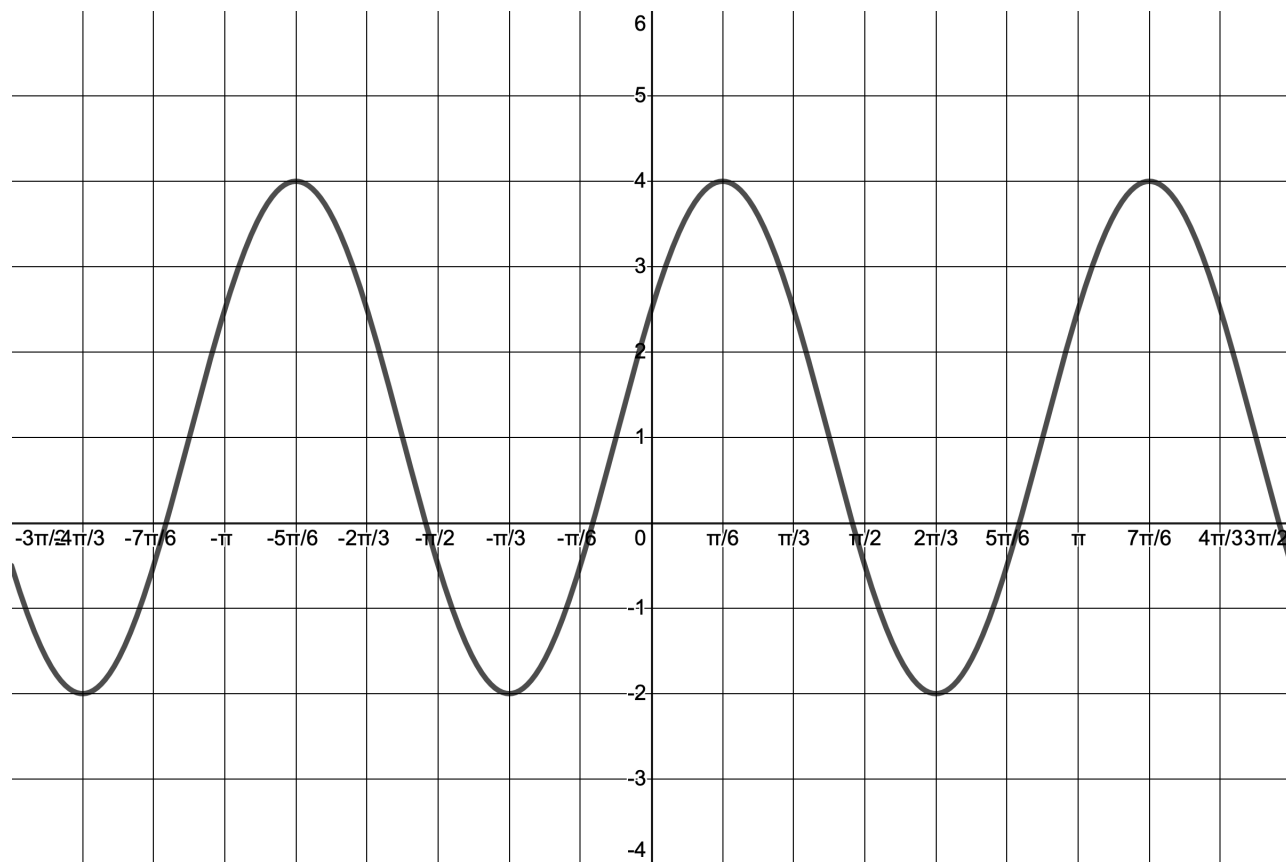
3. [15 marks] Find the exact solution and a two decimal approximation for x where appropriate.

(a) $2 \cos(2x) + 8 \cos(x) + 5 = 0$ for $0 \leq x < 2\pi$ [5 marks] Exact answers only.

(b) $2 \log_2(x + 1) - \log_2(x + 4) = 2$ [5 marks]

(c) $5^x - 2(5^{-x}) = 6$ [5 marks]

4. [5 marks] Below is the graph of a cosine curve whose equation is of the form $y = 3 \cos(2x + \mathbf{c}) + 1$ for some real positive constant $\mathbf{c}$.



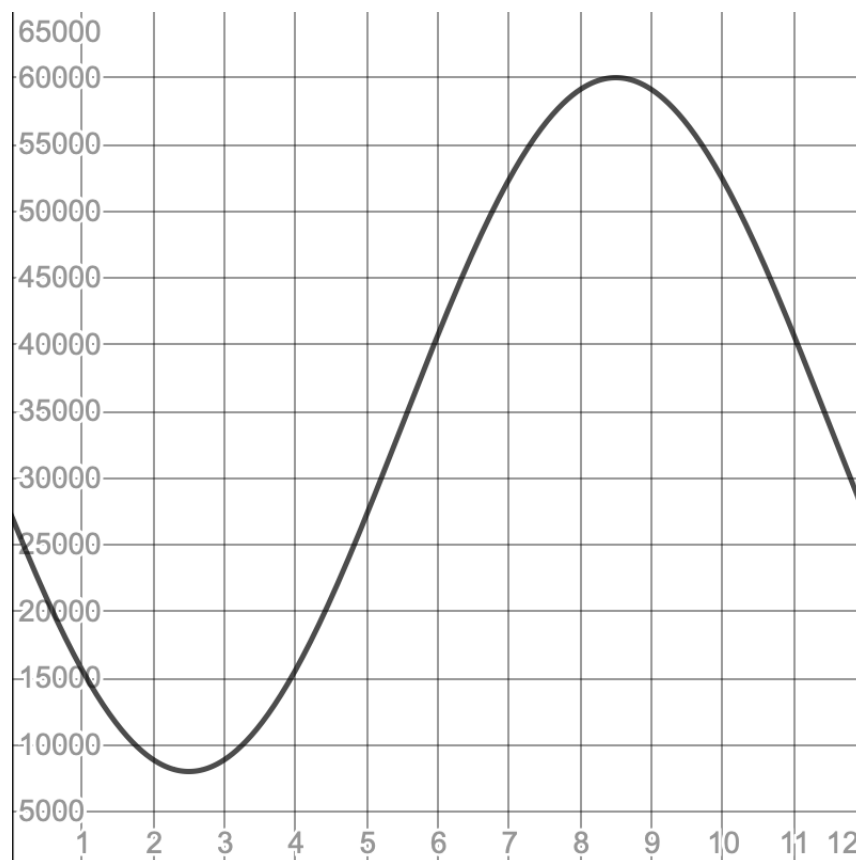
State the amplitude, the period, the phase shift that corresponds to the least positive value $\mathbf{c}$, and find $\mathbf{c}$.

5. [10 marks] A jogger runs at a constant speed of 4 miles per hour in the direction of S 20° E for 30 minutes and then in the direction of N 40° E for the next 15 minutes. Find the straight-line distance from the end point to the starting point of the jogger's course. *Approximate the answer to 2 decimal places. You must include a sketch in your solution.*

6. [10 marks] The flow rate (or water discharge rate) at the mouth of the Orinoco River in South America may be approximated by:

$$F(t) = 26,000 \sin\left(\frac{\pi}{6}(t - 5.5)\right) + 34,000,$$

where t is the time in months and $F(t)$ is the flow rate in m^3/sec . The graph of $y = F(t)$ for $0 \leq t \leq 12$ is provided below.



Find the exact time interval during one year when the flow rate is less than 21,000. *Your answer must be exact, i.e. do not just estimate the answer from the graph.*

7. [7 marks] According to a logistic model of population growth, the earth's population P (in billions) after t years from now is projected to be given by:

$$P = \frac{Ce^{0.05t}}{4 + e^{0.05t}}.$$

(a) If the earth's population now is 7.8 billions, find the value of C . [2 marks]

(b) Using the value of C from part (a), determine how long it will take the earth's population to reach 9 billions. *Approximate the answer to the nearest year.* [5 marks]

8. [4 marks] Let $x > 0$. Write the expression as an algebraic expression in x :

$$\cot \left(\arccos \left(\frac{x}{x+1} \right) \right)$$

9. [5 marks] Given that $f(x) = \sqrt{1 - \ln(x)}$ is a one-to-one function,

(a) Find the domain of f . [3 marks] *Express the domain of f using interval notation.*

(b) Find the inverse function of f . [2 marks] *You do not need to state the domain and range of the inverse function.*

10. [15 marks] Consider the rational function:

$$f(x) = \frac{(3x - 2)^2(3x + 1)}{9x^3 + 18x^2 - x - 2}.$$

- (a) Show that $x + 2$ is a factor of $9x^3 + 18x^2 - x - 2$, and find the factored form of $f(x)$.
Double-check your factoring! [3 marks]

Using the factored form in part (a), identify the following features of the graph $y = f(x)$:

- (b) x -intercept(s): [1 mark]

- (c) y -intercept: [1 mark]

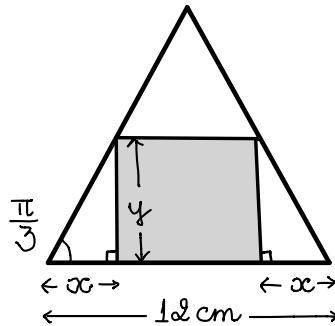
- (d) hole(s): [1 mark]

(e) an equation of the horizontal asymptote (if any): [2 marks]

(f) equations of all vertical asymptotes (if any): [3 marks]

(g) the interval(s) on which $f(x) \leq 0$: [4 marks]

11. [10 marks] In an equilateral triangle whose sides are 12 cm, an inscribed rectangle is constructed as shown in the diagram.



- (a) Express the length y of the rectangle as a function of x , i.e. find $y(x)$. [3 marks]
- (b) Express the area A of the rectangle (the shaded area) as a function of x , i.e. find $A(x)$. What value of x yields the maximum area? And what is the maximum area of the rectangle then? [4 marks]
- (c) If the shaded area is $10\sqrt{3} \text{ cm}^2$, then what are the possible value(s) of x ? [3 marks]

12. [9 marks] Determine whether each of the following statement is True or False.

- If it is True, explain your reasoning.
- If it is False, provide a counter-example and show why it is false.

(a) $\arcsin(\sin(x)) = x$ for every real number x .

(b) If a rational function f is undefined at $x = a$, then $x = a$ is a vertical asymptote of the graph $y = f(x)$.

(c) $\log_{\sqrt{e}}(x) = 2 \ln(x)$ for every $x > 0$.