

## FINAL EXAM

| 1. | X | P(X) | X P(X) | X <sup>2</sup> P(X) |
|----|---|------|--------|---------------------|
|    | 0 | .27  | 0      | 0                   |
|    | 1 | .28  | 0.28   | 0.28                |
|    | 2 | .20  | 0.4    | 0.8                 |
|    | 3 | .15  | 0.45   | 1.35                |
|    | 4 | .08  | 0.32   | 1.28                |
|    | 5 | .02  | 0.1    | 0.5                 |

(a) Mean,  $\mu = \sum X P(X) = 0 + 0.28 + 0.4 + 0.45 + 0.32 + 0.1$   
 $= 1.55$

$$\sigma = \sqrt{\sum X^2 \cdot P(X) - \mu^2}$$

$$= \sqrt{4.21 - 1.55^2} = 1.344$$

(b)  $P(X \geq 1) = 1 - P(X=0) = 1 - 0.27$   
 $= 0.73$

2.  $p = 0.99$ ,  $n = 15$ ,  $q = 1 - 0.99 = 0.01$

(a)  $P(X=13) = \binom{n}{x} p^x \cdot q^{n-x} = \binom{15}{13} \cdot 0.99^{13} \cdot 0.01^2$

$$= 0.0092$$

(b)  $P(14 \text{ or } 15) = \binom{15}{14} \cdot 0.99^{14} \cdot 0.01^1 + \binom{15}{15} \cdot 0.99^{15} \cdot 0.01^0$

$$= 0.1303 + 0.8601 = 0.9904$$

$$(c) P(X < 11) = 1 - P(X \geq 11)$$

$$1 - (P(X=11) + P(X=12) + P(X=13) + P(X=14) + P(X=15))$$

$$= 1 - 0.99999973$$

$$= 0.00000028$$

$$3. \quad p = 0.6$$

$$n = 1000$$

$$\text{Mean, } \mu_x = np = 1000 \times 0.6 = 600$$

$$\sigma_x^2 = np(1-p) = 1000 \times 0.6(1-0.6) = 240$$

$$\sigma_x = \sqrt{240} = 15.492$$

$$4. \quad p = 0.4, \quad n = 14, \quad q = 0.6$$

$$np = 0.4 \times 14 = 5.6 \geq 5, \quad nq = 8.4 \geq 5$$

$$(a) P(X = 7)$$

$$\mu = np = 14 \times 0.4 = 5.6$$

$$\sigma = \sqrt{npq} = \sqrt{5.6 \times 0.6} = 1.83303$$

$$P(X=7) = P(6.5 < X < 7.5)$$

$$Z = \frac{X - \mu}{\sigma}$$

$$P\left(\frac{6.5 - 5.6}{1.83303} < Z < \frac{7.5 - 5.6}{1.83303}\right)$$

$$P(0.491 < Z < 1.037)$$

$$= 0.8501 - 0.6883 = 0.1618$$

$$(b) P(X < 6) = P(X < 5.5)$$

$$P\left(Z < \frac{5.5 - 5.6}{1.83303}\right)$$

$$P(Z < -0.055) = 0.4781$$

$$(c) P(X > 9) = P(X > 9.5)$$

$$= P\left(Z > \frac{9.5 - 5.6}{1.83303}\right)$$

$$P(Z > 2.128) = 0.0167$$

$$5. \quad p = \frac{1}{5} = 0.2, \quad n = 8, \quad P = \frac{1}{5} = 0.2$$

$$(a) P(X = 7) = \binom{8}{7} \cdot 0.2^7 (1-0.2)^1 = 0.000082$$

$$(b) P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7) + P(X=8)$$

$$= 0.0459 + 0.0092 + 0.0011 + 0.000082 + 0.0000026$$

$$= 0.0563$$

$$(c) P(X < 3) = P(X=0) + P(X=1) + P(X=2)$$

$$= 0.1678 + 0.3356 + 0.2936$$

$$= 0.7969$$

$$(d) P(X = 0) = \binom{8}{0} \times 0.2^0 \times (1-0.2)^{8-0}$$

$$= 0.1678$$

$$(e) P(X \geq 1) = 1 - P(X=0) = 1 - 0.1678$$

$$= 0.8322$$

$$6. \quad p = 0.54, \quad n = 12, \quad q = 0.46$$

$$np = 6.48 \geq 5$$

$$nq = 5.52 \geq 5$$

$$\mu = np = 12 \times 0.54 = 6.48$$

$$\sigma = \sqrt{npq} = \sqrt{6.48 \times 0.46} = 1.7265$$

$$(a) \quad P(X=6) = P(5.5 < X < 6.5)$$

$$= P\left(\frac{5.5 - 6.48}{1.7265} < z < \frac{6.5 - 6.48}{1.7265}\right)$$

$$= P(-0.56 < z < 0.011)$$

$$= 0.5044 - 0.2877$$

$$= 0.2167$$

$$(b) \quad P(X < 3) = P(X < 2.5)$$

$$= P\left(z < \frac{2.5 - 6.48}{1.7265}\right)$$

$$= P(z < -2.305) = 0.0106$$

$$(c) \quad P(X \geq 9) = P(X > 8.5)$$

$$= P\left(z > \frac{8.5 - 6.48}{1.7265}\right)$$

$$= P(z > 1.17) = 0.121$$

$$(d) \quad P(X=12) = P(11.5 < X < 12.5)$$

$$= P\left(\frac{11.5 - 6.48}{1.7265} < z < \frac{12.5 - 6.48}{1.7265}\right)$$

$$= P(2.91 < z < 3.49)$$

$$= 0.9998 - 0.99982$$

$$= 0.0016$$

$$(e) P(X=0) = P(-0.5 < X < 0.5)$$

$$= P\left(\frac{-0.5 - 6.48}{1.7265} < Z < \frac{0.5 - 6.48}{1.7265}\right)$$

$$= P(-4.04 < Z < -3.46)$$

$$= 0.0002$$

$$7. p = 0.03, n = 40, q = 0.97$$

$$P(X=0) + P(X=1)$$

$$= \binom{40}{0} \times 0.03^0 \times 0.97^{40} + \binom{40}{1} \times 0.03^1 \times 0.97^{39}$$

$$= 0.2957 + 0.3658$$

$$= 0.6615$$

$$8. \mu_{38} = 38.5 \quad X_{39} = 6$$

$$\text{New mean} = \frac{\mu_{38} \times 38 + 6}{38 + 1} = \frac{5 \times 38 + 6}{39} = 5.0256$$

$$9. \mu_{84} = 17 \quad \mu_{85} = 10$$

$$X_{85} = 85(10) - 84(17) = -518$$

$$10. (a) \text{ Mean, } \bar{x} = \frac{1+2+2+3+3+3+4+4+4+4}{10} = 3$$

$$(b) \text{ Median} = 3$$

$$(c) \text{ Mode} = 4$$

$$(d) \text{ Midrange} = \frac{\text{Max} + \text{Min}}{2} = \frac{4 + 1}{2} = 1.5$$

$$(e) \text{ Range} = \text{Max} - \text{Min} = 4 - 1 = 3$$

$$(f) \text{ Variance} = 1.1111$$

$$(g) \text{ std. dev} = \sqrt{1.1111} = 1.0541$$

$$11. \text{Weighted Mean} = \frac{\sum x_i \cdot w_i}{w_i}$$

$$= \frac{7 \times 10 + 8 \times 15 + 9 \times 12}{10 + 15 + 12}$$

$$= 8.054$$

$$12. C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$C.V_1 = \frac{6}{30} \times 100 = 20\%$$

$$C.V_2 = \frac{5}{25} \times 100 = 20\%$$

The variation is equal

$$13. Z = \frac{x - \mu}{\sigma}$$

$$(a) Z = \frac{75 - 85}{6} = -1.67$$

$$(b) Z = \frac{100 - 115}{9} = -1.67$$

Their relative position is the same

$$14. (a) P(-2 < Z < 0) = 0.5 - 0.0228$$

$$= 0.4772$$

$$(b) P(-0.25 < Z < 0.25) = 0.5987 - 0.4013$$

$$= 0.1974$$

$$15 (a) P(Z < 2.16) = 0.9846$$

$$(b) P(Z > -2.17) = 1 - 0.985 = 0.015$$

$$(c) P(Z > 0) = 1 - 0.5 = 0.5$$

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|       | a    | b    | c    | Total |
|-------|------|------|------|-------|
| M     | 2000 | 3000 | 4000 | 9000  |
| F     | 5050 | 1750 | 3800 | 10600 |
| Total | 7050 | 4750 | 7800 | 19600 |

$$(a) P(F) = \frac{10600}{19600} = \frac{53}{98}$$

$$(b) P(c) = \frac{7800}{19600} = \frac{39}{98}$$

$$(c) P(\text{male from town c}) = \frac{4000}{19600} = \frac{10}{49}$$

$$(d) P(\text{male from b or female from a}) = \frac{3000}{19600} + \frac{5050}{19600} = \frac{23}{56}$$

$$(e) P(\text{male}) \text{ or } P(\text{town c}) \text{ or } P(\text{male and c})$$

$$= \frac{9000}{19600} + \frac{7800}{19600} - \frac{4000}{19600} = \frac{33}{49}$$

$$(f) P(\text{male}) \text{ or } P(\text{town c}) \text{ but not both}$$

$$= \frac{9000}{19600} + \frac{7800}{19600} = \frac{6}{7}$$

$$(g) P(M \text{ or } F) = \frac{9000}{19600} + \frac{10600}{19600} = 1$$

$$17. \mu = 53, \sigma = 4$$

$$Z = \frac{x - \mu}{\sigma}$$

$$(a) P(x > 59) = P\left(z > \frac{59 - 53}{4}\right)$$

$$= P(z > 1.5) = 0.0668$$

$$(b) P(x < 45) = P\left(z < \frac{45 - 53}{4}\right) = P(z < -2) = 0.0228$$

$$(c) P(50 < x < 55) = P\left(\frac{50 - 53}{4} < z < \frac{55 - 53}{4}\right)$$

$$= P(-0.75 < z < 0.5) = 0.6915 - 0.2266$$

$$= 0.4649$$

$$\begin{aligned}
 (d) P(58 < X < 62) &= P\left(\frac{58-53}{4} < Z < \frac{62-53}{4}\right) \\
 &= P(1.25 < Z < 2.25) \\
 &= 0.9878 - 0.8944 \\
 &= 0.0934
 \end{aligned}$$

18  $\mu = 49, \sigma = 8$

$$\begin{aligned}
 (a) P(X \leq 55) &= P\left(Z \leq \frac{55-49}{8}\right) \\
 &= P(Z \leq 0.75) = 0.7734
 \end{aligned}$$

$$\begin{aligned}
 (b) P(X \geq 62) &= P\left(Z \geq \frac{62-49}{8}\right) \\
 &= P(Z > 1.625) = 1 - 0.9521
 \end{aligned}$$

$$\begin{aligned}
 (c) P(46 < X < 54) &= P\left(\frac{46-49}{8} < Z < \frac{54-49}{8}\right) \\
 &= P(-0.375 < Z < 0.625) \\
 &= 0.734 - 0.3538 \\
 &= 0.3802
 \end{aligned}$$

$$\begin{aligned}
 (d) P(X < 49) &= P\left(Z < \frac{49-49}{8}\right) \\
 &= P(Z < 0) = 0.5
 \end{aligned}$$

19 (a) 25<sup>th</sup> Percentile

$$Z = \frac{X - \mu}{\sigma}$$

$$P(Z < k) = 0.25$$

$$Z = -0.67$$

$$-0.67 = \frac{X - 49}{8}$$

$$X = (-0.67)(8) + 49 = 43.64$$

(b) 40<sup>th</sup> percentile

$$P(Z < k) = 0.4$$

$$Z = -0.25$$

$$-0.25 = \frac{x - 49}{8}$$

$$x = (-0.25)(8) + 49 = 47$$

(c) 80<sup>th</sup> percentile

$$P(Z < k) = 0.8$$

$$Z = 0.84$$

$$0.84 = \frac{x - 49}{8}$$

$$x = (0.84)(8) + 49 = 55.74$$