

**MATH 107 QUIZ 5**

Oct 2020

Instructor: Dr. Fitzpatrick

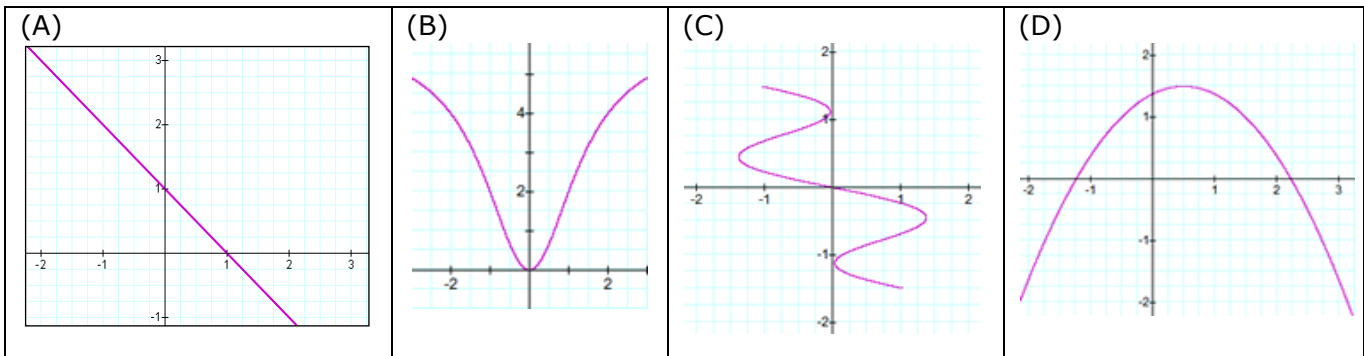
**NAME:** \_\_\_\_\_

I have completed this assignment myself, working independently and not consulting anyone except the instructor.

**INSTRUCTIONS**

- The quiz is worth 100 points, and there is an extra credit opportunity at the end. There are 10 problems. This quiz is *open book* and *open notes*. This means that you may refer to your textbook, notes, and online classroom materials, but ***you must work independently and may not consult anyone*** (and confirm this with your submission). You may take as much time as you wish, provided you turn in your quiz no later than **Monday, October 28**.
- **Show work/explanation where indicated. Answers without any work may earn little, if any, credit.** You may type or write your work in your copy of the quiz, or if you prefer, create a document containing your work. Scanned work is acceptable also. **In your document, be sure to include your name and the assertion of independence of work.**
- General quiz tips and instructions for submitting work are posted in the Quizzes module.
- If you have any questions, please contact me by e-mail.

1. (4 pts) Which of these graphs represent a one-to-one function? **Answer(s):** \_\_\_\_\_  
(no explanation required.) (There may be more than one graph that qualifies.)



2. (6 pts) Based on data about the growth of a variety of ornamental cherry trees, the following logarithmic model about these trees was determined:

$$h(t) = 6.108 \ln(t) + 6.099, \text{ where } t = \text{age of tree in years and } h(t) = \text{height of tree, in feet.}$$

(Note that "ln" refers to the natural log function) (explanation optional)

Using the model,

(a) At age 3 years, how tall is this type of ornamental cherry tree, to the nearest tenth of a foot?

(b) At age 12 years, how tall is this type of ornamental cherry tree, to the nearest tenth of a foot?

3. (4 pts) Convert to a logarithmic equation:  $5^x = 645$ . (no explanation required)

2. \_\_\_\_\_

A.  $\log_5 645 = x$

B.  $\log_x 645 = 5$

C.  $\log_x 5 = 645$

D.  $\log_5 x = 645$

4. (8 pts) Solve the equation. Check all proposed solutions. Show work in solving and in checking, and state your final conclusion.

$$\sqrt{x - 1} = 3 - x$$

5. (8 pts)

(a)  $\log_{36} 36^{216} = \underline{\hspace{2cm}}$  (fill in the blank)

(b) Let  $x = \log_{36} \frac{1}{216}$  State the exponential form of the equation.

(c) Determine the numerical value of  $\log_{36} \frac{1}{216}$ , in simplest form. Work optional.

6. (10 pts) Let  $f(x) = 2x^2 - x - 9$  and  $g(x) = 4x + 1$

(a) Find the composite function  $(f \circ g)(x)$  and simplify the results. Show work.

(b) Find  $(f \circ g)(-2)$  . Show work.

7. (16 pts) Let  $f(x) = \frac{4x+2}{3x-6}$

(a) Find  $f^{-1}$ , the inverse function of  $f$ . **Show work.**

(b) What is the domain of  $f$ ? What is the domain of the inverse function?

(c) What is  $f(3)$ ?  $f(3) = \underline{\hspace{2cm}}$  work/explanation optional

(d) What is  $f^{-1}(\underline{\hspace{2cm}})$ , where the number in the blank is your answer from part (c)? work/explanation optional

8. (18 pts) Let  $f(x) = e^{x-1} + 4$ .

Answers can be stated without additional work/explanation.

(a) Which describes how the graph of  $f$  can be obtained from the graph of  $y = e^x$ ? Choice: \_\_\_\_\_

- A. Shift the graph of  $y = e^x$  to the left by 1 unit and up by 4 units.
- B. Shift the graph of  $y = e^x$  to the right by 1 unit and up by 4 units.
- C. Reflect the graph of  $y = e^x$  across the  $x$ -axis and shift up by 4 units.
- D. Reflect the graph of  $y = e^x$  across the  $y$ -axis and shift up by 4 units.

(b) What is the domain of  $f$ ?

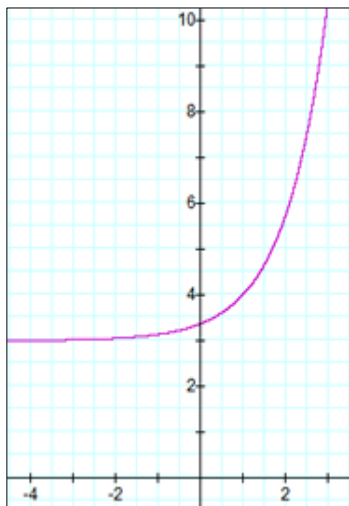
(c) What is the range of  $f$ ?

(d) What is the horizontal asymptote?

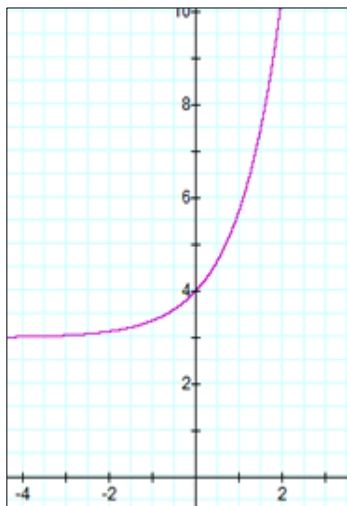
(e) What is the  $y$ -intercept? State the approximation to 2 decimal places (i.e., the nearest hundredth).

(f) Which is the graph of  $f$ ?

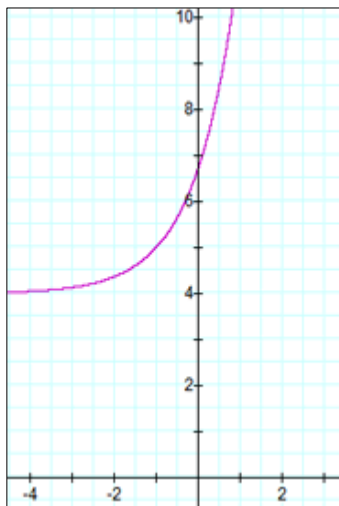
GRAPH A



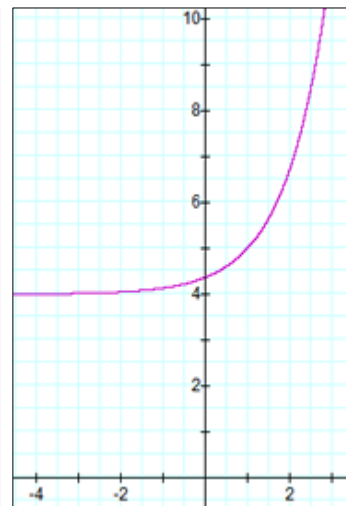
GRAPH B



GRAPH C



GRAPH D

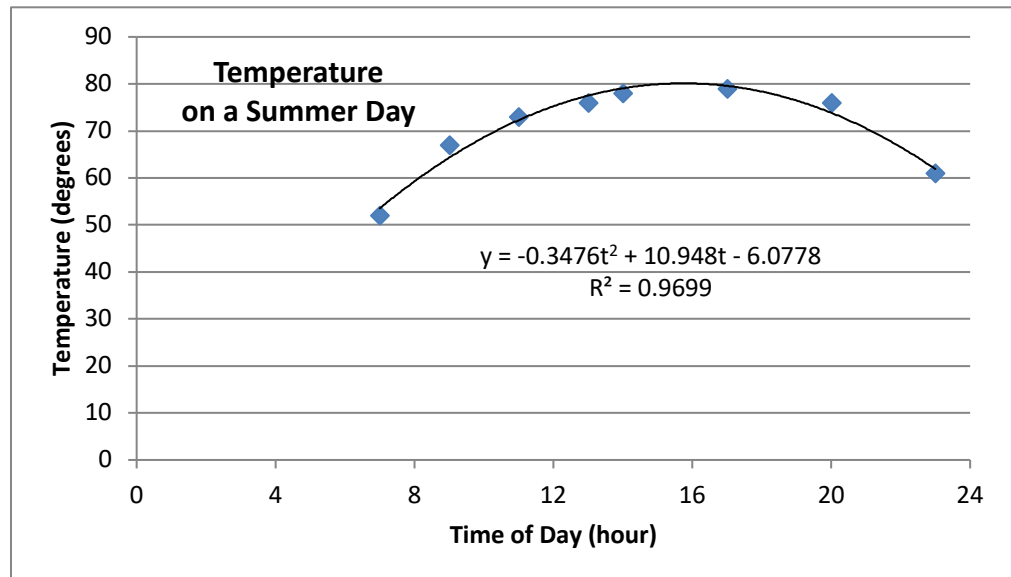


**NONLINEAR MODELS** - For the latter part of the quiz, we will explore some nonlinear models.

**9. (16 pts) QUADRATIC REGRESSION**

**Data:** On a particular summer day, the outdoor temperature was recorded at 8 times of the day, and the following table was compiled. A scatterplot was produced and the parabola of best fit was determined.

| $t =$<br>Time<br>of day<br>(hour) | $y =$ Outdoor<br>Temperature<br>(degrees F.) |
|-----------------------------------|----------------------------------------------|
| 7                                 | 52                                           |
| 9                                 | 67                                           |
| 11                                | 73                                           |
| 13                                | 76                                           |
| 14                                | 78                                           |
| 17                                | 79                                           |
| 20                                | 76                                           |
| 23                                | 61                                           |



**Quadratic Polynomial of Best Fit:**

$$y = -0.3476t^2 + 10.948t - 6.0778 \text{ where } t = \text{Time of day (hour) and } y = \text{Temperature (in degrees)}$$

REMARKS: The times are the hours since midnight. For instance, 7 means 7 am, and 13 means 1 pm.

(a) Using algebraic techniques we have learned, find the **maximum temperature predicted by the quadratic model** and find the **time when it occurred**. Report the time to the nearest quarter hour (i.e., \_\_:00 or \_\_:15 or \_\_:30 or \_\_:45). (For instance, a time of 18.25 hours is reported as 6:15 pm.) Report the maximum temperature to the nearest tenth of a degree. **Show algebraic work.**

(b) Use the quadratic polynomial to estimate the outdoor temperature at 6:30 am, to the nearest tenth of a degree. (work optional)

(c) Use the quadratic polynomial  $y = -0.3476t^2 + 10.948t - 6.0778$  together with algebra to **estimate the time(s) of day when the outdoor temperature  $y$  was 77 degrees.**

That is, solve the quadratic equation  $77 = -0.3476t^2 + 10.948t - 6.0778$  .

**Show algebraic work in solving.** State your results clearly; report the time(s) to the nearest quarter hour.

## 10. (10 pts) + (extra credit at the end) EXPONENTIAL REGRESSION

**Data:** A cup of hot coffee was placed in a room maintained at a constant temperature of 65 degrees, and the coffee temperature was recorded periodically, in Table 1.

**TABLE 1**

| $t$ = Time Elapsed (minutes) | $C$ = Coffee Temperature (degrees F.) |
|------------------------------|---------------------------------------|
| 0                            | 166.0                                 |
| 10                           | 140.5                                 |
| 20                           | 125.2                                 |
| 30                           | 110.3                                 |
| 40                           | 104.5                                 |
| 50                           | 98.4                                  |
| 60                           | 93.9                                  |

### REMARKS:

Common sense tells us that the coffee will be cooling off and its temperature will decrease and approach the ambient temperature of the room, 65 degrees.

So, the temperature difference between the coffee temperature and the room temperature will decrease to 0.

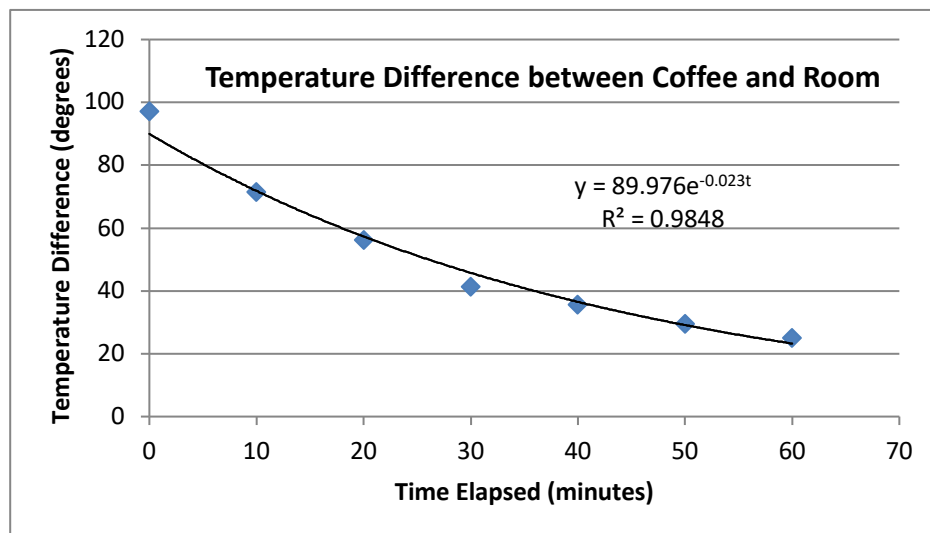
We will fit the temperature difference data (Table 2) to an exponential curve of the form  $y = A e^{-bt}$ .

Notice that as  $t$  gets large,  $y$  will get closer and closer to 0, which is what the temperature difference will do.

So, we want to analyze the data where  $t$  = time elapsed and  $y = C - 65$ , the temperature *difference* between the coffee temperature and the room temperature.

**TABLE 2**

| $t$ = Time Elapsed (minutes) | $y = C - 65$ Temperature Difference (degrees F.) |
|------------------------------|--------------------------------------------------|
| 0                            | 97.0                                             |
| 10                           | 71.5                                             |
| 20                           | 56.2                                             |
| 30                           | 41.3                                             |
| 40                           | 35.5                                             |
| 50                           | 29.4                                             |
| 60                           | 24.9                                             |



**Exponential Function of Best Fit (using the data in Table 2):**

$y = 89.976 e^{-0.023t}$  where  $t$  = Time Elapsed (minutes) and  $y$  = Temperature Difference (in degrees)

(a) Use the exponential function to estimate the temperature difference  $y$  when 25 minutes have elapsed. Report your estimated temperature difference to the nearest tenth of a degree. (explanation/work optional)

(b) Since  $y = C - 65$ , we have coffee temperature  $C = y + 65$ . Take your difference estimate from part (a) and add 65 degrees. Interpret the result by filling in the blank:

When 20 minutes have elapsed, the estimated coffee temperature is \_\_\_\_\_ degrees.

(c) Suppose the coffee temperature  $C$  is 95 degrees. Then  $y = C - 65 =$         degrees is the temperature *difference* between the coffee and room temperatures.

(d) Consider the equation         $= 89.976 e^{-0.023t}$  where the        is filled in with your answer from part (c).

**EXTRA CREDIT (6 pts):**

Show algebraic work to solve this part (d) equation for  $t$ , to the nearest tenth. Interpret your results clearly in the context of the coffee application. [Use additional paper if needed]