Question 1

1. $a_n = \left(-\frac{2}{3}\right)^{n-1}$

$n = 1$

$$\left(-\frac{2}{3}\right)^{1-1} = \left(-\frac{2}{3}\right)^{0}$$

Ans = 1

$n = 2$

$$\left(-\frac{2}{3}\right)^{2-1}$$

$$= \left(-\frac{2}{3}\right)$$

$n = 3$ $\left(-\frac{2}{3}\right)^{3-1}$

$$= \frac{4}{9}$$

$n = 4$ $\left(-\frac{2}{3}\right)^{4-1}$

$$= -0.2963$$

$n = 5$ $\left(-\frac{2}{3}\right)^{5-1}$

$$= 0.1975.$$

QUESTION 1

Question 3.

3 $b_n = \frac{(-1)^n x^n}{n}$

$n=1 = \frac{(-1)^1 x^1}{1}$

$= -x$

$n=2 = \frac{(-1)^2 x^2}{2}$

$= \frac{x^2}{2}$.

$n=3 \quad \frac{(-1)^3 x^3}{3}$

$= \frac{-x^3}{3}$

$n=4 \quad \frac{(-1)^4 \cancel{x}^4}{4}$

$= \frac{x^4}{4}$

$n=5 \quad \frac{(-1)^5 x^5}{5}$

$= \frac{-x^5}{5}$.

Question 4

$b_n = \frac{(-1)^{2n+1} x^{2n+1}}{(2n+1)!}$

$n=1$

$\frac{(-1)^3 x^3}{3!}$

$= \frac{-x^3}{6}$

~~n=2~~ $n=2$

$\frac{(-1)^5 x^5}{5!}$

$= \frac{-x^5}{120}$

$n=3$

$\cancel{\frac{(-1)^7 x^7}{8!}}$

$n=3$

$\frac{(-1)^7 x^7}{7!}$

$= \cancel{\frac{-x^7}{40320}} \; \frac{-x^7}{5040}$

$n=4$

$\frac{(-1)^9 x^9}{9!}$

$\frac{-x^9}{362880}$

$n=5$

$\frac{(-1)^{11} x^{11}}{39916800}$

Question 5-

Simplify $= \dfrac{11! \cdot 4!}{4! \cdot 7!}$

$= \dfrac{11! \cdot \cancel{4!}}{\cancel{4!} \; 7!}$

$\dfrac{11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$

$= 7920$

Question 6

Simplify $\frac{n!}{(n+1)!}$

$$\frac{\cancel{n}(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{\cancel{n}(n+1)(n-1)(n-2)\cdots 3\times 2\times 1}$$

$= n+1$

Question 7

$$\frac{2n!}{(n-1)!}$$

$$\frac{2 \cdot n \cancel{(n-1)}\cancel{(n-2)} \cdots \cancel{2} \cdot \cancel{1}}{\cancel{(n-1)} \cdot \cancel{(n-2)} \cancel{2} \cdot \cancel{1}}$$

Ans = $2n$.

Question 8.

2, 5, 10, 17, 26

3 5 7 9

2 2 2.

$a_n = an^2 + bn + c$

Let 1^{st} team $= x$

2^{nd} team $= y$

2^{nd} difference $= z$

$2a = z$

$3a + b = y - x$

$a + b + c = x$

$2a = 2$

$3a + b = 3$

$a + b + c = 2.$

$2a = 2$

$a = \frac{2}{2} = 1$

$a = 1$

$3a + b = 3$

$3(1) + b = 3$

$b = 0$

$a + b + c = 2$

$(1) + (0) + c = 2$

$c = 2 - 1$

$c = 1$

Expression

$n^{th} = (1)n^2 + 0(n) + 1$

n^{th} team $= n^2 + 1$

Question 9.

first term $a_1 = 5000$

Common diff. $d = -100$

$a_n = a_1 + d(n-1)$

$= a_1 + (n-1)d.$

$= 5000 + (n-1) - 100$

$5000 + 100 - 100n.$

$$\frac{1\cancel{00}n}{1\cancel{00}} = \frac{51\cancel{00}}{1\cancel{00}}$$

$n = 51$

$a_n = 5000 + (51-1) - 100$

Question 10

Geometric: $a_1 = 4$, $a_{k+1} = \frac{1}{2} a_k$.

$a_n = a_1 r^{n-1}$

$a_{k+1} = a_1 (r)^{(k+1)-(1)}$

$\frac{1}{4} \times \frac{1}{2} a_k = \frac{4 \cancel{r}^k}{4}$

$\sqrt[k]{\frac{1}{8} a_k} = \sqrt[k]{r^k}$

$r = \sqrt[k]{\frac{1}{8} a_k}$

$r = \left[\frac{1}{8} a_k\right]^{1/k}$

$a_1 = \frac{1}{\left[\frac{1}{8} a_k\right]^{1/k}} =$

$a_n = \frac{1}{\cancel{\left[\frac{1}{8} a_k\right]^{1/k}}} \times \left(\cancel{\left[\frac{1}{8} a_k\right]^{1/k}}\right)^{n-1}$

$a_{\cancel{n} \cancel{=} \cancel{n-1}}$

$a_n = 1^{n-1}$

Question 11

Use Sigma notation

$$\frac{2}{3(1)+1} + \frac{2}{3(2)+1} + \ldots \frac{2}{3(12)+1}$$

$$\sum_{1}^{12} \frac{2}{3n+1} = \frac{2}{4} + \frac{2}{7} + \frac{2}{10} + \frac{2}{13} + \frac{2}{16} + \frac{2}{19} + \frac{2}{22} + \frac{2}{25} + \frac{2}{28} + \frac{2}{31} + \frac{2}{34} + \frac{2}{37}$$

Question 12

Use sigma notation

$$\sum_{2}^{1/128} 1/4n = 2 + 1/2 + 1/8 + 1/32 + 1/128$$

Question 13

$$\sum_{n=1}^{7} (8n-5) = 8(1)-5 + 8(2)-5 + 8(3)-5 + 8(4)-5 + 8(5)-5 + 8(6)-5 + 8(7)$$

$3 + 11 + 19 + 27 + 35 + 43 + 51$

$= 189$

Question 14

$$\sum_{n=1}^{8} 24(1/6)^{n-1} = 24(1/6)^{1-1} + 24(1/6)^{2-1} + 24(1/6)^{3-1} + 24(1/6)^{4-1} + 24(1/6)^{5-1} + 24(1/6)^{6-1} +$$

$$24(1/6)^{7-1} + 24(1/6)^{8-1}$$

$$24 + 4 + 2/3 + 1/9 + 1/54 + 1/324 + 1/1944 + 1/11664$$

$$= 28\ \frac{9331}{11664}$$

Question 15

$$5 - 2 + \frac{4}{5} - \frac{8}{25} + \frac{16}{125}$$

$$\frac{625 - 250 + 100 - 40 + 16}{125}$$

$$\text{Ans} = 3\frac{76}{125}.$$

Question 16

Using induction

$$3+6+9+\dots+3n = \frac{3n(n+1)}{2}$$

Add n+1 both sides.

$$3+6+9+\dots+3n+n+1 = \frac{3n(n+1)}{2} + n+1$$

Having one denomiaratoa

Since $(3+6+9+\dots+3n) = \frac{3n(n+1)}{2}$

Thus $$\frac{3n(n+1)}{2} + \frac{2(n+1)}{2} = \frac{3n(n+1)+2(n+1)}{2}$$

$$\frac{3n(n+1)+2(n+1)}{2} = \frac{3n(n+1)+2n(n+1)}{2}$$

Using numenatons.

$$3n^2+3+2n+2 = 3n^2+3+2n+2$$

There two equal equations bothsides hence.

$3+6+9+\dots+3n = \frac{3n(n+1)}{2}$ has been prooven.

Question 17

Binomial Theorem $(2a-5b)^4$ Expand 1 4 6 4 1 - Pascal -

$(2a-5b)^4 = 16a^4 + 4(2a)^3(-5b) + 6(2a)^2(-5b)^2 + (2a)4(-5b)^3 + (-5b)^4$

$16a^4 - 160a^3b + 600a^2b^2 - 1000ab^3 + 625b^4$

$(2a-5b)^4 = 16a^4 - 160a^3b + 600a^2b^2 - 1000ab^3 + 625b^4$

Question 18

$_9C_3$

$_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{9!}{6!\,3!}$

$\frac{\cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7 \cdot \cancel{6!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6!}}$

$= 84$

Question 19

$_{20}C_3$

$_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{20!}{17!\,3!}$

$\frac{\cancel{20}^{10} \cdot 19 \cdot \cancel{18}^{6} \cdot \cancel{17!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{17!}}$

$= 1140$

Question 20

$_9P_2$

$_nP_r = \frac{n!}{(n-r)!} = \frac{9!}{7!}$

$\frac{9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$

$= 72$

Question 21

$_{70}P_3$

$_nP_r = \frac{n!}{(n-r)!} = \frac{70!}{67!}$

$\frac{70 \cdot 69 \cdot 68 \cdot \cancel{67!}}{\cancel{67!}}$

$= 328440$

Question 22

$4 \cdot {}_nP_3 = {}_{n+1}P_4$

$$4 \cdot \frac{n!}{n-3} = \frac{(n+1)!}{(n+1-4)!}$$

$$4 \cdot \frac{\cancel{n}(\cancel{n}-1)(\cancel{n}-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = \frac{(n+1)\cancel{n}(\cancel{n}-1)(\cancel{n}-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

$4 = n+1$

$n = 4-3$

$n = 3.$

Question 23

Letters $= 26$

Numbers $= 10.$

Digits needed $= 3.$

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{26!}{(26-3)!}$$

$$\frac{26 \times 25 \times \overset{4}{\cancel{24}} \times \cancel{23!}}{\cancel{3 \times 2 \times 1} \times \cancel{23!}}$$

$= 2600$ plates.

Question 25

Red Cards -7

Diamonds - 13

Hearts - 13

Deck of Cards = 52.

N° of favourable outcomes = ~~52~~ 13*13 = 26.

Total n° of favourable outcome = 52.

$$\text{Probability} = \frac{\text{Number of favourable Outcome}}{\text{Total number of favourable outcome}}$$

$$\frac{26}{52} = \frac{1}{2}$$

Question 24

Selected are = 4

Class of = 25.

The four students can be selected from a class of 25 in $_{25}C_4$ ways

$$nC_r = \frac{n!}{(n-r)!r!} \quad \frac{25!}{(25-4)!4!}$$

$$= \frac{25 \times \cancel{24} \times 23 \times 22 \times \cancel{21!}}{4 \times \cancel{3} \times 2 \times 1 \times \cancel{21!}}$$

= 12650 ways.

Question 26

$P_A = n(A) | n(S)$

Number of fauouraable $n(A) = 6$

~~Number of~~ Total number of Outcome $= 11$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{11}$$

Question 27

a. Both numbers are odd.

Odd numbers → 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39
41, 43, 45, 47, 49, 51, 53, 55, 57, 59

The space is $n(S) = 60$

Number of events $n(E) = 30$

The probability $= \frac{n(E)}{n(S)} = \frac{\cancel{30}\,1}{\cancel{60}\,2}$

$= \frac{30}{60} = \frac{1}{2}$.

Probability of both the numbers are odd

$P(E) \cdot P(E) = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

b. both Number are less than 12

Numbers less than 12 → 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Space is $n(S) = 11$

Event $n(E) = 1$

Probability of one number $= \frac{n(E)}{n(S)} = \frac{1}{11}$

Probability of both

$P(E) \cdot P(E) = \frac{1}{11} \times \frac{1}{11}$

$= \frac{1}{121}$

c. Same number is chosen twice?

Space is $n(S) = 120$.

Event $n(E) = 2$.

Probability same number chosen twice $= \frac{n(E)}{n(S)} = \frac{\cancel{2}\,1}{\cancel{120}\,60} = \frac{1}{60}$.

Question 28

Probability of snowing Event 75%

Probability of not snowing Event = 25%

Space n(S) = 100%

Probability of not snowing $\frac{n(E)}{n(S)} = \frac{\cancel{25}\%}{\cancel{100}\%} = \frac{1}{4}$

$= \frac{1}{4}$