Problem 1

$3x^2 + 3y^2 = 96.$

$5x^2 - 5y^2 = 0.$

soln:

$5x^2 = 5y^2.$

$x^2 = y^2$

substituting, we get:

$3y^2 + 3y^2 = 96.$

$\frac{6y^2}{6} = \frac{96}{6} \Leftrightarrow y^2 = 16, \; y = \pm 4$

when $y = +4,$

$3x^2 + 3(4)^2 = 96. \Leftrightarrow 3x^2 + 48 = 96 \Leftrightarrow x^2 = 16$

$x = \pm 4$

$\therefore (4, 4)$ and $(-4, 4)$

when $y = -4$

$3x^2 + 3(-4)^2 = 96 \Leftrightarrow 3x^2 + 48 = 96 \Leftrightarrow x^2 = 16$

$x = \pm 4$

$\therefore (-4, -4) \; (4, -4)$

solutions:

$(4, 4), \; (-4, 4), \; (4, -4), \; (-4, -4)$

Problem 2

Rewrite in standard form.

$x^2 - 10x + y^2 - 4y - 9 = 0.$

soln.

$\Rightarrow x^2 - 10x + 5^2 + y^2 - 4y + 2^2 = 9 + 5^2 + 2^2.$ (completing the square)

$\Rightarrow (x-5)^2 + (y-2)^2 = 38$

standard form: $(x-5)^2 + (y-2)^2 = 38$

center: $(h, k) = (5, 2)$

radius $= r = \sqrt{r^2} = \sqrt{38}.$

Points on the circle: $r = \sqrt{38}$

, The points are: $(5, 2+\sqrt{38})$, $(5, 2-\sqrt{38})$, $(5+\sqrt{38}, 2)$, $(5-\sqrt{38}, 2)$

Problem 3

$0 = 9x^2 - 6x + 65.$

soln.

$a = 9 \quad b = -6, \quad c = 65.$

$$x = \frac{-b \pm \sqrt{\cancel{4ac}\, b^2 - 4ac}}{2a}.$$

$$= \frac{6 \pm \sqrt{36 - (4 \times 9 \times 65)}}{18} = \frac{6 \pm \sqrt{-2304}}{18}.$$

$$= \frac{1}{3} \pm \frac{i\sqrt{2304}}{18} = \frac{1}{3} \pm \frac{8}{3}i$$

Problem 4.

$y = 28x - 2x^2 - 96.$

soln.

x - intercept. At x-intercept, $y = 0$.

$\therefore \; -2x^2 + 28x - 96 = 0.$

$a = -2 \quad b = 28 \quad c = 96$

$$x = \frac{-28 \pm \sqrt{28^2 \cancel{+} 4 \times 2 \times 96}}{-4} = \frac{28 \pm \sqrt{16}}{-4}$$

$x = 6$ or $x = 8$.

x intercept: $(6,0)$, $(8,0)$

y intercept, $x = 0$.

$y = -96 \quad (=) \quad (0, -96)$

vertex of parabola : $\frac{dy}{dx} = 0.$

$\frac{dy}{dx} = 28 - 4x = 0.$

$\frac{4x}{4} = \frac{28}{4} \quad x = 7$

when $x = 7$, $y = 28(7) - 2(7)^2 - 96. = 2.$

$= (7, 2)$

The correct graph is Graph E.

Problem 5.

Simplify: $\cancel{\frac{9}{y^2} \div}$ $\dfrac{\frac{9}{y^2} + \frac{1}{y}}{\frac{81}{y^2} - 1}$

soln.

Combine $\frac{81}{y^2} - 1 = \frac{81 - y^2}{y^2}$

and

$\frac{9}{y^2} + \frac{1}{y} \Rightarrow \frac{9 + y}{y^2}$

The eqn becomes:

$\cancel{\dfrac{\frac{81 - y^2}{y^2}}{9+y}}$ $\dfrac{\frac{9+y}{y^2}}{\frac{81-y^2}{y^2}} = \frac{9+y}{\cancel{y^2}} \times \frac{\cancel{y^2}}{81-y^2}$

$= \cancel{9+y} \times \frac{1}{(9-y)\cancel{(9+y)}} = \frac{1}{9-y}$ or $\frac{-1}{y-9}$

Problem 6

$7^x = 2398$.

soln.

a) Introducing ln both sides.

$\ln 7^x = \cancel{2398} \ln 2398$

$x \ln 7 = \ln 2398$

$x = \frac{\ln 2398}{\ln 7}$

$x = 3.999$

Problem 7

$\theta = -\frac{4}{3}\pi \qquad -(\pi + \frac{\pi}{3})$

a) Quadrant. $\quad -\frac{4}{3} \times 180 = -240°$

$-\frac{4}{3}\pi$ is in Quadrant $\cancel{4}$2 $\qquad \theta$.

b) Reference angle.

$-\frac{4}{3}\pi \qquad 2\pi - \frac{4}{3}\pi = \frac{2}{3}\pi$

The reference angle is $\pi - \frac{2}{3}\pi = \frac{1}{3}\pi$.

c) Intersect the unit circle

$x = -\cos\theta$ (since $r = 1$)

$x = \cos(\pi/3) = -\frac{1}{2}$.

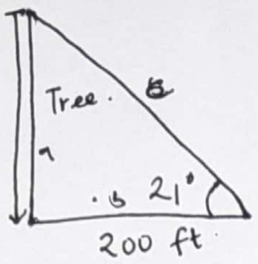
$y = \sin\theta$

$= \sin(\pi/3) = \frac{\sqrt{3}}{2}$.

Therefore the intercept (x, y)

$= (-\frac{1}{2}, \frac{\sqrt{3}}{2})$

Problem 8



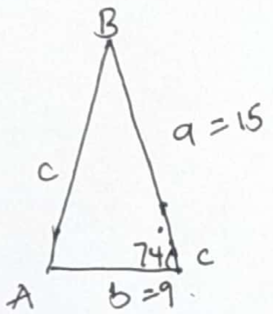
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$

$$\tan 21 = \frac{h}{200}$$

$$h = 200 \times \tan 21$$

$$= 76.77 \approx 77 \text{ feet.}$$

problem 9.



Using cos rule, $c^2 = a^2 + b^2 - 2ab\cos C$

$$c^2 = 15^2 + 9^2 - 2(9)(15)\cos 74.$$

$$= 225 + 81 - 270\cos 74.$$

$$= 306 - 74.42$$

$$c^2 = 231.57$$

$$= 15.218$$

Using sin rule,

$$\frac{a}{\sin A} = \frac{c}{\sin C} \quad (=) \quad \frac{15}{\sin A} = \frac{15.2177}{\sin 74.}$$

$$\frac{15}{\sin A} = 15.83094$$

$$\sin A = \frac{15}{15.83094}$$

$$\angle A = 71.35°$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}. \quad (=) \quad \frac{9}{\sin B} = \frac{15.2177}{\sin 74.}$$

$$\sin B = \frac{9}{15.83094}$$

$$\angle B = 34.65$$

Problem 10

Find solns on $[0, 2\pi]$

$\tan(x) + 1 = 0.$

$\tan(x) = -1$

$\tan x = -1 \iff x = \tan^{-1}(-1)$

Quadrant 2 and 4.

$x = \frac{3}{4}\pi, \frac{7}{4}\pi$