MBA 803 – FUNDAMENTALS OF FINANCE

Time Value of Money Practice Problems – Solutions

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Note: Some of the problems below came from *Corporate Finance*, 6th edition by Ross, Westerfield, and Jaffee.

In solving the following problems, I strongly encourage you to write out time lines to keep the cash flows straight. In addition, you may find it useful to use the mathematical formulae to help you understand the intuition behind the problems.

1) Calculate the following future values and answer the associated questions:

   a) Compute the future value of $500 compounded annually for 6 years at 5 percent.

      $FV = 500 \times 1.05^6 = \$670.05.$

   b) Compute the future value of $500 compounded annually for 12 years at 5 percent.

      $FV = 500 \times 1.05^6 = \$897.93.$

   c) Why is the interest earned in part *b* not twice the interest earned in part *a*?

      The total interest earned in the first case is $170.05, while it is $397.93. The reason the second figure is not twice the first (despite twice the time over which it is earned) is because of compound interest. That is, each year when you receive an interest payment it is added to your principal balance to increase the amount of interest you earn during the following year.

   d) Compute the future value of $500 compounded *quarterly* for 6 years at 5 percent.

      $FV = 500 \times (1 + 0.05/4)^{6\times4} = 500 \times 1.0125^{24} = \$673.68.$

   e) Explain briefly the difference between your answers for parts *a* and *d*.

      In part *a* you only begin earning "interest on your interest" and the end of each year. In part *d*, this happens every quarter. Thus, your interest begins compounding sooner in part *d*, leading to a larger total future value.

2) Calculate the present values of the following cash flows assuming a 12 percent discount rate, compounded annually:

   a) $1,000 to be received in five years.

      $PV = 1{,}000 \div 1.12^5 = \$567.43.$

   b) $45,000 to be received in three years.

      $PV = 45{,}000 \div 1.12^3 = \$32{,}030.11.$

   c) $14,000 to be received in twelve years.

      $PV = 14{,}000 \div 1.12^{12} = \$3{,}593.45.$

3) Would you rather receive $4,000 today or $5,000 five years from now if your discount rate is 6 percent?

   The present value of $5,000 to be receive five years from now is PV = $5{,}000 \div 1.06^5 = \$3{,}736.29$. This is less than $4,000, so you would rather receive the $4,000 today.

4) How long will it take for a $3,000 investment to grow to $10,000 if invested at 5 percent interest

   a) compounded annually?

      $10{,}000 = 3{,}000 \times 1.05^T \Rightarrow T = \ln(10{,}000 \div 3{,}000) \div \ln(1.05) = 24.68$ years.

   b) compounded quarterly?

      $10{,}000 = 3{,}000 \times (1 + 0.05/4)^T \Rightarrow T = \ln(10{,}000 \div 3{,}000) \div \ln(1.0125) = 96.92$ quarters, or 24.23 years.

5) What annual interest rate would you have to earn to provide the same return as an account that paid 4 percent interest compounded quarterly?

   $EAR = (1 + 0.04 / 4)^4 - 1 = 4.06\%$.

6) You have been offered an investment that will pay you $500 per month for the next ten years. This investment will cost you $30,000 to undertake.

   a) If your required rate of return is 12 percent (compounded monthly), should you make this investment?

      The net present value of this investment is

$$
\begin{aligned}
NPV &= -30{,}000 + \sum_{t=1}^{120} \frac{500}{(1+0.12)^t} \\
&= -30{,}000 + 500\left(1 - \frac{1}{(1+0.12/12)^{120}}\right)\frac{1}{0.12/12} \\
&= -30{,}000 + 34{,}850.26 \\
&= 4{,}850.26
\end{aligned}
$$

      Because this is positive, you should make the investment.

   b) At this discount rate, what is the most you would be willing to pay for this investment?

      The most you would be willing to pay is the present value of the future cash flows, or $34,850.26.

7) You want to lease a car for three years. The original value of the car is $30,000 and at the end of three years you expect it to be worth $10,000. If the required interest rate is 6.50 percent (compounded monthly), how large will your monthly lease payment be?

This trick with this type of problem is to find the monthly payment that will make the present value of all future value of all future cash flows (including the $10,000 residual value) equal to the $30,000 cost of the automobile.

In a financial calculator, set the payments to occur at the beginning of the period. You can then enter P/Y = 12, N = 36, I = 6.50, PV = 30,000, FV = – 10,000, and solve for PMT = – 663.55.

8) What is the present value of a bond that promises to pay $100 per year forever into the future if your discount rate is 15 percent, compounded annually?

This bond is simply a perpetuity, so PV = 100 / 0.15 = $666.67.

9) Consider a stock that will begin paying annual dividends three years from today. Its initial dividend is expected to be $2 per share, and this divided is expected to grow by 4 percent per year forever into the future. What is the value of this stock if your discount rate is 13 percent?

This stock will be a growing perpetuity, but note that it does not begin making payments for three years. Its value *two* years from today is 2 ÷ (0.13 – 0.04) = $22.22. Its value today is therefore 22.22 ÷ 1.13^2 = $17.40.

10) You would like to make monthly payments into a savings account to fund a three-year trip around the world that you plan to take in five years. You will need $40,000 per year to make this trip, which you expect to withdraw on a quarterly basis over the three year trip (12 withdrawals of $10,000 each). Your savings account pays 7.50 percent interest, compounded monthly. Your payments will be made at the end of each month, while your withdrawals during the trip will occur at the beginning of each quarter. How large must your monthly payments be to achieve your goal?

To solve this problem you need to figure out how much money you will need in your account at the beginning of your trip. The quarterly withdrawals you will make from your savings account occur at the beginning of each quarter, so this is an annuity due.

This is easiest to solve in your financial calculator. Set your calculator to solve for an annuity due (i.e., set payments to be made at the beginning of the period). Then set P/Y =4 and C/Y = 12, for quarterly payments with monthly compounding. Finally, enter N = 12, I = 7.5, PMT = 10,000, FV = 0 and solve for PV = – 108,501.99. This is how much must be in your account at the beginning of your trip. In other words, it is how much you must save over the next five years.

The next step is to figure out how large of payments will be necessary to save this amount. Set P/Y =12 and C/Y = 12 to have monthly payments with monthly compounding. Also, set your calculator back to solve for an ordinary annuity (payments at the end of the period). Next enter N = 60, PV = 0, FV = 108,501.99, and solve for PMT = $1,496.02.

11) When Marilyn Monroe died, ex-husband Joe-DiMaggio vowed to place fresh flowers on her grave every Sunday as long as he lived. A bunch of fresh flowers that the former baseball player thought appropriate for the star cost about $5 when she died in

1962. Based on actuarial tables, “Jolton’ Joe” could expect to live for 30 years after the actress died. Assume that the stated annual interest rate, compounded weekly, is 10.4 percent. Also, assume that the rate of inflation is 3.9 percent per year, when expressed as a stated annual inflation rate, compounded weekly. Assuming that each year has exactly 52 weeks, what is the present value of this commitment?

This is simply a growing annuity problem, albeit with the complication of weekly payments and compounding. To solve it, simply convert the discount rate and the growth rate into weekly periodic rates and then use the growing annuity formula.

$$PV = C\left[1-\left(\frac{1+g/m}{1+r/m}\right)^{T\times m}\right]\frac{1}{r/m-g/m}$$

$$= 5\left[1-\left(\frac{1+0.039/52}{1+0.104/52}\right)^{30\times 52}\right]\frac{1}{0.104/52-0.039/52}$$

$$= 5[1-0.14266]\frac{1}{0.00125}$$

$$= \$3{,}429.38\,.$$