

33190 Mathematical Modelling for Science: Mastery Test 4

45 minutes – Competency requirement – 70%

INCLUDE WORKING OUT – or no marks will be awarded

Integral Table and formula sheet is attached

Name: _____ Student Number: _____

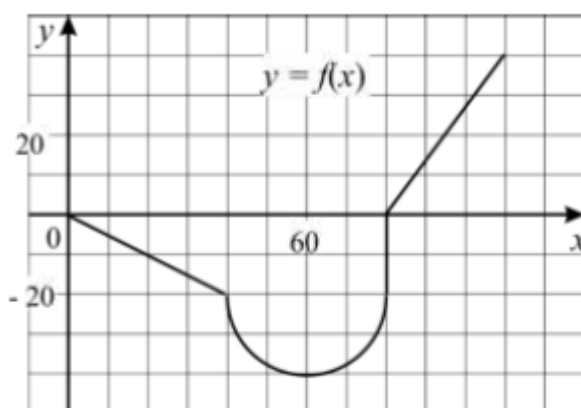
Date: _____ Lab Tutor: _____

Start: _____ Finish: _____ Mark: _____/40

1. - **8 points** (2+3+3)

Use the graph of $f(x)$ to evaluate each integral.

$$\int_0^{40} f(x) dx$$



$$\int_{40}^{80} f(x) dx$$

$$\int_0^{110} f(x) dx$$

2. – 6 points (3+ 3)

(a) Evaluate the integral:

$$\int_0^{\sqrt{3}/2} \frac{8}{\sqrt{1-t^2}} dt$$

(b) Find the root mean square (RMS) of

$$f(x) = e^{x/2}$$

on the interval $0 \leq x \leq 4$.

3. - 6 points

Evaluate the following integral by using partial fractions.

$$\int \frac{x+4}{(x-1)(x+3)} dx$$

4. - 10 points (5+5)

(a) Evaluate the following integral by an appropriate substitution:

$$\int x^4(5 + x^5)^5 dx$$

(b) Evaluate the integral:

$$\int_0^{\pi/8} \cos^2 4x dx$$

5. – Find the value of this integral 5 points

$$\int \cos x \sin^3 x dx$$

6. -- Find the value of this integral 5 points

$$\int x e^{5x} dx$$

Table of Integrals

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln|x + \sqrt{x^2 - a^2}| + C_2$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln(x + \sqrt{x^2 + a^2}) + C_2$$

$$\int \frac{dx}{1 - x^2} = \tanh^{-1} x + C_1 = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C_2 \quad \int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| < a. \\ \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| > a. \end{cases}$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \tanh x \, dx = \ln \cosh x + C$$

Formula sheet

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$A(x) = \int_a^x f(t) \, dt$$

$$\int_a^b f(t) \, dt = [F(x)]_a^b = F(b) - F(a)$$

$$\bar{y} = \frac{1}{(b-a)} \int_a^b f(x) \, dx$$

$$RMS = \sqrt{\frac{1}{b-a} \int_a^b (f(x))^2 \, dx}$$

$$\int_a^b f(x) \, dx \approx \frac{h}{2} [f(a) + f(b)]$$

$$W = \int_a^b kx \, dx$$

$$\int uv' \, dx = uv - \int v u' \, dx$$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$