

33130 Mathematical Modelling 1: Mastery Test 4

(second attempt)

45 minutes – Competency requirement – 70%

INCLUDE WORKING OUT – or no marks will be awarded

Integral Table and formula sheet is attached

Name: _____ Student Number: _____

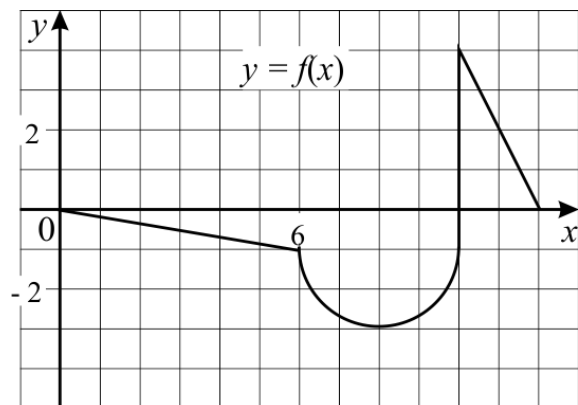
Date: _____ Lab Tutor: _____

Start: _____ Finish: _____ Mark: _____ /40

1. - **8 points** (2+3+3)

Use the graph of $f(x)$ to evaluate each integral.

$$\int_0^6 f(x) dx$$



$$\int_6^{10} f(x) dx$$

$$\int_0^{12} f(x) dx$$

2. – 6 points (3+ 3)

(a) Evaluate the integral:

$$\int_{1/2}^{\sqrt{2}/2} \frac{7}{\sqrt{1-t^2}} dt$$

(b) Find the root mean square (RMS) of

$$f(x) = e^{x/4}$$

on the interval $0 \leq x \leq 16$.

3. - 6 points

Evaluate the following integral by using partial fractions.

$$\int \frac{x+3}{(x+4)(x-2)} dx$$

4. - 10 points (5+5)	
(a) Evaluate the following integral by an appropriate substitution: $\int 6x^5(5 + x^6)^7 dx$	(b) Evaluate the integral: $\int_0^{\pi/2} \sin^2 4x dx$
5. – Find the value of this integral 5 points $\int \sin^4 x \cos x dx$	
6. – Find the value of this integral 5 points $\int x \cos(2x) dx$	s

Table of Integrals

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln|x + \sqrt{x^2 - a^2}| + C_2$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln(x + \sqrt{x^2 + a^2}) + C_2$$

$$\int \frac{dx}{1 - x^2} = \tanh^{-1} x + C_1 = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C_2 \quad \int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| < a. \\ \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| > a. \end{cases}$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \tanh x \, dx = \ln \cosh x + C$$

Formula sheet

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$A(x) = \int_a^x f(t) \, dt$$

$$\int_a^b f(t) \, dt = [F(x)]_a^b = F(b) - F(a)$$

$$\bar{y} = \frac{1}{(b-a)} \int_a^b f(x) \, dx$$

$$RMS = \sqrt{\frac{1}{b-a} \int_a^b (f(x))^2 \, dx}$$

$$\int_a^b f(x) \, dx \approx \frac{h}{2} [f(a) + f(b)]$$

$$W = \int_a^b kx \, dx$$

$$\int uv' \, dx = uv - \int v u' \, dx$$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$