

Directions: Read each question carefully. Clearly show answers. Show all work to receive full credit.

1. We will represent the results of six tosses of a coin as an ordered list of length 6 with entries from $\{H,T\}$. How many different results are possible?

2. A password has five numbers, each in the range 1 to 9. How many different passwords are there?

3. A certain state has a simple rule for its license plates: Use two letters followed by four digits. How many possible plates are there?

4. The Greek alphabet has 24 letters. How many four-letter sequences of Greek letters do not consist of four of the same letter?

5. How many variations in first-, second-, and third-place finishes are possible in a 100yard dash with twelve runners

6. In how many ways can six taxis line up in front of a hotel?

7. How many subsets of $\{0, 2, 4, 6, 8, 10\}$ have three elements?

8. A raffle requires the purchase of a ticket that has 4 different numbers taken from $\{1, 2, 3, \dots, 20\}$. The order of the numbers does not matter. How many different raffle tickets are there?

9. Consider $(x + y)^{10}$. When $(x+y)^{10}$ is expanded, what is the coefficient of x^5y^5 ?

10. Let A and B be square matrices of the same size. Which one of the following is true for all A and B? [Select your Answer].

(1) $A+B = B+A$

(2) $A-B = B-A$

(3) $AB = BA$

(4) None of the above.

11. Let: $A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 5 & 2 \\ 0 & -2 \end{bmatrix}$

Compute the indicated operations if possible.

11a. $3A+2D$

11b. $C-B^T$

11c. DC

11d. CC^T

11e. $AD-DA$

12. Use the Gaussian elimination to solve the system :

$$x - 2y + 2z = 3$$

$$2x + 3y - z = 5$$

$$y + z = 5$$

13. Use the matrix equations and inverses to solve the system:

$$2x + y = -1$$

$$3x + y = -3$$

14. Use the Gauss-Jordon method to find the inverse of the matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

15. Compute the determinant using cofactor expansion along any row or column.

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$