

Instructor: Arnold Yim**Name:** _____

Instructions:

1. Exam Time: 120 minutes
2. Work on the exam until 10am. You have until 10:15am to upload your exam back onto Canvas.
3. Put a box around your answers.
4. You must show sufficient work to justify all answers. An answer without any work may not receive any credit.
5. Calculators, notes, books, and videos permitted.

Page	Score	Points Possible
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total		100

1. (10 points) Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ -2 & 4 & 3 & 1 \\ 1 & -2 & -2 & -1 \end{bmatrix}$. Answer the following and box your answers.

- (a) Find a basis for the column space of A .
- (b) Find a basis for the row space of A .
- (c) Find a basis for the null space of A .
- (d) What is the dimension of the null space of A ?
- (e) What is the rank of A ?

2. (10 points) Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \end{bmatrix} \right\}$ be two bases for $\mathbb{R}^2$.

(a) What is $P_{\mathcal{B}}$ (the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis)?

(b) If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, what is $\vec{x}$?

(c) If $\vec{x} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, what is $[\vec{x}]_{\mathcal{B}}$?

(d) What is $P_{\mathcal{C} \leftarrow \mathcal{B}}$ (the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$)?

(e) If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, what is $[\vec{x}]_{\mathcal{C}}$?

- 3.** (10 points) Find the eigenvalues of the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & -2 \end{bmatrix}$. For each eigenvalue, describe the corresponding eigenspace.

4. (10 points) A certain recreation room has two types of games: table tennis and pool. A study finds that 40% of people that play table tennis one day will continue playing table tennis the next day, and 20% of people that play pool one day will play table tennis the next day instead. Suppose that on the first day, half of the people choose to play table tennis and the other half of the people choose to play pool.

(a) What is the stochastic matrix that describes people's choice in recreational games?

(b) What is the initial state vector $\vec{x}_0$?

(c) What percentage of the people are playing pool on the second day?

(d) What percentage of the people are playing table tennis on the third day?

5. (10 points) Let $\vec{u} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Compute the following.

(a) $\vec{u} \cdot \vec{v}$

(b) $\vec{u} \cdot \vec{w}$

(c) $\vec{v} \cdot \vec{w}$

(d) Which pair of vectors are orthogonal?

(e) $\vec{u} \times \vec{v}$

(f) $\vec{v} \times \vec{w}$

6. (5 points) Find the projection of $\vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ onto $\vec{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

7. (5 points) Find the area of the triangle with vertices $A(2, 1, 1)$, $B(4, 0, 3)$, and $C(1, 2, 3)$.

8. (10 points) Solve the following linear system:

$$x_1 - 3x_2 = 5$$

$$-x_1 + x_2 + 5x_3 = 2$$

$$x_2 + x_3 = 0$$

9. (10 points) Determine if the following are linearly independent or dependent.

(a) $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}.$

(b) $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

10. (10 points) Find the inverse of the following matrices.

(a) $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

11. (10 points) Calculate the determinant of the following matrices, then state whether or not the matrix is invertible.

(a)
$$\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$