Math 4
Exam 1

Show your work! Write solutions and supporting work on separate papers. All work, including graphs, should be handwritten. No graphing calculator print outs. Reduce all fractions. Simplify all radicals. Be sure to include units for word problems. All points labeled on graphs should be given as ordered pairs.

1. Simplify the given expression. **No decimals. (20 pts.)**

$$\frac{1}{9}(3x+2)^{\frac{4}{5}}(x-4)^{-\frac{1}{2}} - \frac{1}{6}(3x+2)^{-\frac{1}{5}}(x-4)^{\frac{1}{2}}$$

2. Solve. Express your answer in interval notation. **No decimals. (20 pts.)**

$$\left|\frac{3}{7} - 4x\right| + 5 > 14$$

3. State the domain of the given function using interval notation. Justify your answer. Show how any part of the expression for this function must be restricted in order for a value for the variable, x, to be in the domain of the function. Use a scanning line when solving inequalities. **No decimals. (20 pts.)**

$$f(x) = \frac{\sqrt{(x+10)(x-3)(x-5)(x+2)}}{x-14}$$

4. A spherical balloon has an initial radius of 4 ft. Air is then pumped into this balloon at a constant rate of $\frac{8}{9}\pi$ cubic feet per minute. Express the radius, r, of the balloon as a function of time, t (in minutes), spent pumping air into the balloon. Assume that the difference between the volume of the balloon and the volume of air in the balloon is negligible. **No decimals.** $V = \frac{4}{3}\pi r^3$ **(20 pts.)**

5. Sketch the graph of the given piece-wise function $f(x)$.
   1) Label the point or points where a transition to a different equation occurs.
   2) If a graph has a vertex or inflection point, label it.
   3) Label six additional points with integer coordinates on your graph as follows: two points for $x < -1$, two points for $-1 < x < 3$, and two points for $x > 3$.

   (20 pts.)

$$f(x) = \begin{cases} \frac{2}{3}x + \frac{14}{3}, & \text{if } x \leq -1 \\ -(x-3)^2 + 7, & \text{if } x > -1 \end{cases}$$

6. List the sequence of compressions, stretches, shifts or reflections applied to the function f(x) to obtain the function g(x). Graph g(x).
   1) If g(x) is a cubic function, label the inflection point. If g(x) is an absolute value or quadratic function, label the vertex.
   2) Label a second point with integer coordinates on your graph.
   3) Label a third point using the second point and symmetry on your graph.
   4) State the intercepts as ordered pairs.

   No decimals. Do not graph the parent function $f(x)$.

   (25 pts. each)

a) $f(x) = |x|$ $\qquad$ $g(x) = -7|x+3|+10$

b) $f(x) = x^3$ $\qquad$ $g(x) = \frac{1}{3}(x-2)^3 - 6$

7. List the sequence of compressions, stretches, shifts or reflections applied to the function $f(x) = \sqrt{x}$ to obtain the given function $g(x)$ shown below. Graph $g(x)$.
   1) Label the endpoint on your graph.
   2) Label two points with integer coordinates on your graph.
   3) The point $(4, 2)$ is a point on the graph of $f(x) = \sqrt{x}$. Determine what this point becomes after $f(x)$ is transformed into $g(x)$. Label it on your graph.
   4) State the intercepts as ordered pairs.

   No decimals. Do not graph the parent function $f(x)$. (25 pts.)

$$g(x) = 3\sqrt{6-x} - 5$$