

Mathematics

Name

[Institutional Affiliation(s)]

Question 1

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

Let  $m = \lambda$

$$\text{Det}(A - \lambda I) = 0$$

$$\left| \left( \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{vmatrix} = 0$$

$$= ((1-\lambda)(4-\lambda)) - (-1 \times 2) = 0$$

$$= (1(4-\lambda) - \lambda(4-\lambda)) - (-2) = 0$$

$$= \lambda^2 - 5\lambda + 4 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

Factorising left side of the equation

$$(\lambda - 2)(\lambda - 3) = 0$$

Set the factors equal to zero

$$\lambda - 2 = 0 \text{ or } \lambda - 3 = 0$$

$$\lambda = 2 \text{ or } \lambda = 3$$

But  $m = \lambda$

then  $m = 2 \text{ or } m = 3$



## Question 2.

$$B = \begin{bmatrix} 10 & -6 & -4 \\ -6 & 11 & 2 \\ -4 & 2 & 6 \end{bmatrix}$$

$$B = \lambda I$$

Let  $\lambda$  be eigen value of matrix  $B$

$$\Leftrightarrow (B - \lambda I) \vec{v} = 0$$

$$\Leftrightarrow |B - \lambda I| = 0$$

$$\left( \begin{bmatrix} 10 & -6 & -4 \\ -6 & 11 & 2 \\ -4 & 2 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det \begin{pmatrix} 10-\lambda & -6 & -4 \\ -6 & 11-\lambda & 2 \\ -4 & 2 & 6-\lambda \end{pmatrix} = 0$$

$$\lambda^3 - \left( \begin{matrix} \text{Sum of diagonal} \\ \text{elements} \end{matrix} \right) \lambda^2 + \left( \begin{matrix} \text{Sum of diagonal} \\ \text{minors} \end{matrix} \right) \lambda - |B| = 0$$

$$\lambda^3 - 27\lambda^2 + 180\lambda - 324 = 0$$

Factoring left side of the equation

$$(\lambda - 3)(\lambda - 6)(\lambda - 18)$$

$$\lambda - 3 = 0, \text{ or } \lambda - 6 = 0, \text{ or } \lambda - 18 = 0$$

$$\lambda = 3 \text{ or } \lambda = 6 \text{ or } \lambda = 18$$

Finding eigen vectors

$$\text{Eigen values} = (3, 6, 18)$$

Finding eigenvectors  
when  $\lambda = 3$

$$\begin{bmatrix} 7 & -6 & -4 \\ -6 & 8 & 2 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7v_1 - 6v_2 - 4v_3 = 0$$

$$-6v_1 + 8v_2 + 2v_3 = 0$$

$$v_1 = \begin{vmatrix} -6 & -4 \\ 8 & 2 \end{vmatrix} = -12 + 32 = 20$$

$$v_2 = -\begin{vmatrix} 7 & -4 \\ -6 & 2 \end{vmatrix} = -(14 - 24) = 10$$

$$v_3 = \begin{vmatrix} 7 & -6 \\ -6 & 8 \end{vmatrix} = 56 - 36 = 20$$

$$V_1 = \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix} \text{ or } \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

when  $\lambda = 6$

$$\begin{bmatrix} 4 & -6 & -4 \\ -6 & 5 & 2 \\ -4 & 2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4v_1 - 6v_2 - 4v_3 = 0$$

$$-6v_1 + 5v_2 + 2v_3 = 0$$

$$v_1 = \begin{vmatrix} -6 & -4 \\ 5 & 2 \end{vmatrix} = -12 + 20 = 8$$

$$v_2 = -\begin{vmatrix} 4 & -4 \\ -6 & 2 \end{vmatrix} = -(8 - 24) = 16$$

$$V_3 = \begin{vmatrix} 4 & -6 \\ -6 & 5 \end{vmatrix} = 20 - 36 = -16$$

$$V_2 = \begin{bmatrix} 8 \\ 16 \\ -16 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

When  $\lambda = 18$

$$\begin{bmatrix} -8 & -6 & -4 \\ -6 & -7 & 2 \\ -4 & 2 & -12 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-8v_1 - 6v_2 - 4v_3 = 0$$

$$-6v_1 - 7v_2 + 2v_3 = 0$$

$$v_1 = \begin{vmatrix} -6 & -4 \\ -7 & 2 \end{vmatrix} = -12 - 28 = -40$$

$$v_2 = - \begin{vmatrix} -8 & -4 \\ -6 & 2 \end{vmatrix} = -(-16 - 24) = \cancel{40} 40$$

$$v_3 = \begin{vmatrix} -8 & -6 \\ -6 & -7 \end{vmatrix} = 56 - 36 = 20$$

$$V_3 = \begin{bmatrix} 40 \\ 40 \\ 20 \end{bmatrix} \text{ or } \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Two eigenvectors are said to be orthogonal iff their dot product is zero.

$$\vec{v}_1 \cdot \vec{v}_2 = \langle 2, 1, 2 \rangle \cdot \langle 1, 2, -2 \rangle$$

$$= 2 + 2 - 4 = 0$$

Hence  $\vec{v}_1$  and  $\vec{v}_2$  are orthogonal.

$$\vec{v}_1 \cdot \vec{v}_3 = \langle 2, 1, 2 \rangle \cdot \langle 2, 2, 1 \rangle$$

$$= \cancel{4 + 2 + 2} \neq 0$$

$$= -4 + 2 + 2 = 0$$

Hence  $\vec{v}_1$  and  $\vec{v}_3$  are orthogonal.

$$\vec{v}_2 \cdot \vec{v}_3 = \langle 1, 2, -2 \rangle \cdot \langle -2, 2, 1 \rangle$$

$$= -2 + 4 - 2 = 0$$

Hence  $\vec{v}_2$  and  $\vec{v}_3$  are orthogonal.

Question 3

$$81 = \lambda$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$0 = 81 - 21 - 21 - 21 = 0$$

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$$\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = 0$$