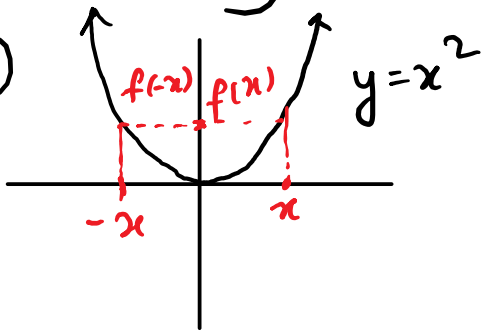


Even/Odd functions

Symmetry in a graph:

①



Test for symmetry
Replace $x \rightarrow -x$

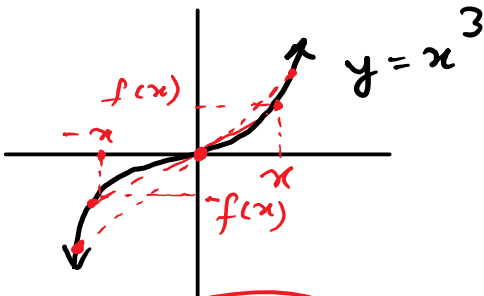
$$f(x) = f(-x)$$

$f(x)$ is called **EVEN** function
 $f(x)$ has y -axis symmetry

$$x = 5 \rightarrow f(5) = (5)^2 = \underline{\underline{25}}$$

$$x = -5 \rightarrow f(-5) = (-5)^2 = \underline{\underline{25}}$$

②



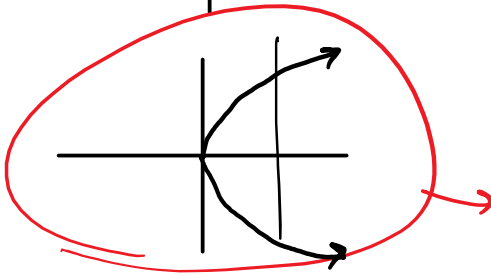
$$x = 5 \rightarrow f(5) = (5)^3 = \underline{\underline{125}}$$

$$x = -5 \rightarrow f(-5) = (-5)^3 = \underline{\underline{-125}}$$

Replace $x \rightarrow -x$

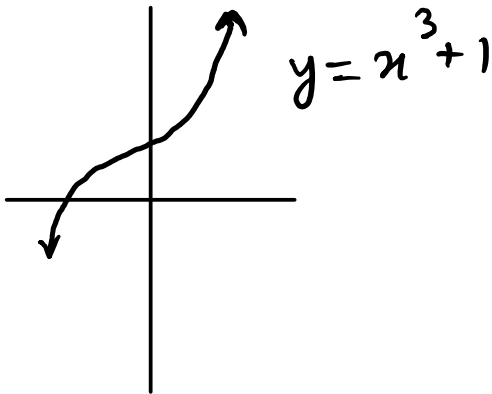
then $\underline{\underline{f(x) = -f(-x)}}$

$f(x)$ are called **ODD** functions



Not
a function

③



$$x = 5 \rightarrow (5)^3 + 1 = \underline{\underline{126}}$$

$$x = -5 \rightarrow (-5)^3 + 1 = \underline{\underline{-124}}$$

No symmetry!

DQ

Find the difference quotient for $f(x) = \sqrt{x+1}$.

Solution:

Rationalizing

$$\begin{aligned}
 \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\
 &= \frac{(\overset{a}{\sqrt{x+h+1}} - \overset{b}{\sqrt{x+1}}) \cdot (\overset{a}{\sqrt{x+h+1}} + \overset{b}{\sqrt{x+1}})}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
 &= \frac{(\overset{a^2}{x+h+1} - \overset{b^2}{x+1})}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
 &= \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{x+h+1 - x-1}{h(\sqrt{\quad} + \sqrt{\quad})} \\
 &= \boxed{\frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}}
 \end{aligned}$$

$g(x) = 2\sqrt{x-1}$

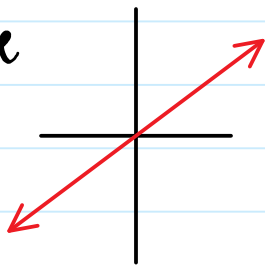
DQ: $\frac{g(x+h) - g(x)}{h} = \frac{[2\sqrt{x+h-1}] - [2\sqrt{x-1}]}{h}$

$$\begin{aligned}
 &= 2 \left\{ \frac{\overset{a}{\sqrt{x+h-1}} - \overset{b}{\sqrt{x-1}}}{h} \right\} \times \frac{\overset{a}{\sqrt{x+h-1}} + \overset{b}{\sqrt{x-1}}}{\sqrt{x+h-1} + \sqrt{x-1}} \\
 &= 2 \left\{ \frac{(\sqrt{x+h-1})^2 - (\sqrt{x-1})^2}{h} \right\} = \frac{2[x+h-1 - x+1]}{h[\quad]}
 \end{aligned}$$

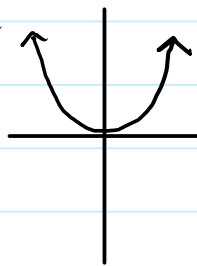
...

Library of functions

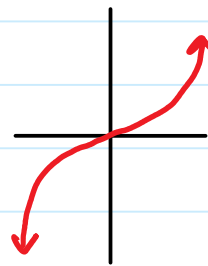
① $y = x$



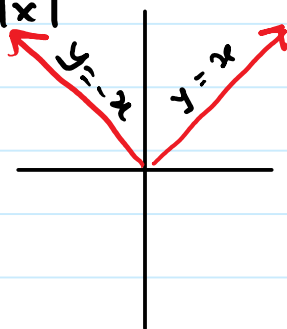
② $y = x^2$



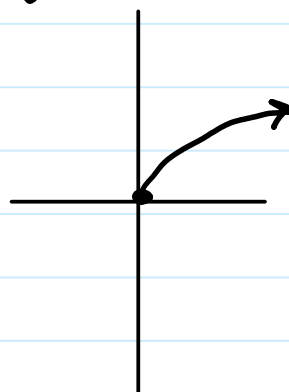
③ $y = x^3$



④ $y = |x|$



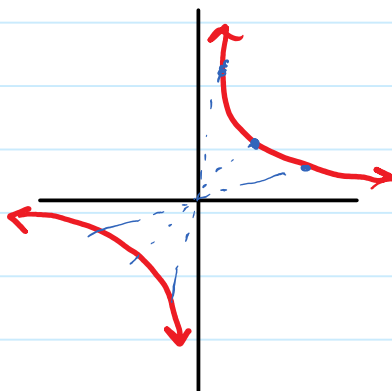
⑤ $y = \sqrt{x}$



⑥ $y = \sqrt[3]{x}$



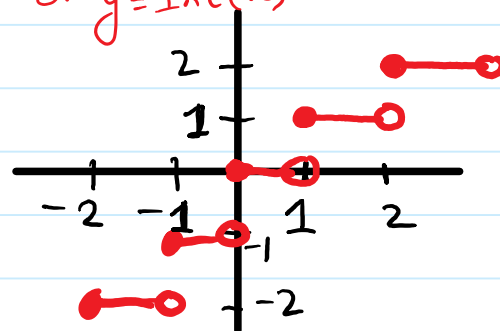
⑦ $y = \frac{1}{x}$



⑧ greatest Integer function
(step function)

$y = [x]$

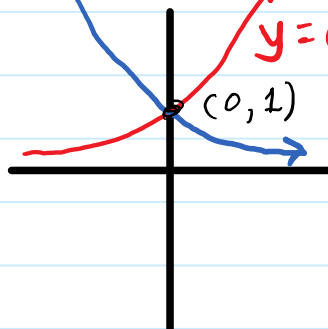
or $y = \text{Int}(x)$



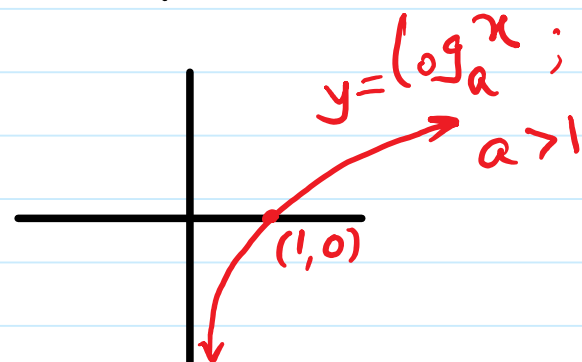
$y = a^x; 0 < a < 1$

$y = a^x; a > 1$

9)



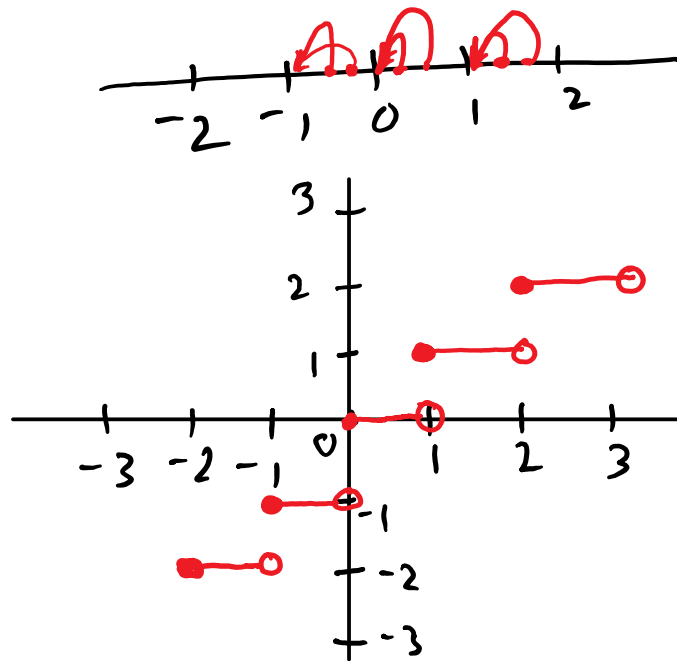
10)



$$y = [x]$$

$y = [x] \text{ or } y = \text{Int}(x)$ (no decimal)
 "The output (y) is the greatest integer
not exceeding the input (x)"

x	$y = [x]$
1.25	1
1.50	1
1.75	1
1.99	1
2	2
2.1	2
2.9	2
0.25	0
.75	0
-.25	-1
-.5	-1



Step function

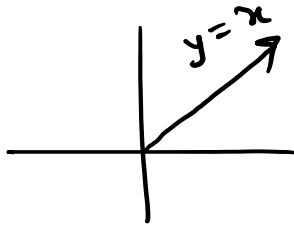
□ 2.4 Constructing The graph of piecewise functions.

(domain is split up)

$$f(x) = \begin{cases} \boxed{f_1(x)} & ; D_1(x) \\ \boxed{f_2(x)} & ; D_2(x) \\ \boxed{f_3(x)} & ; D_3(x) \end{cases}$$

ex:

$$y = |x|$$



x	y
1	1
2	2
3	3
0	0
-1	1
-2	2
-3	3

 $y = x$

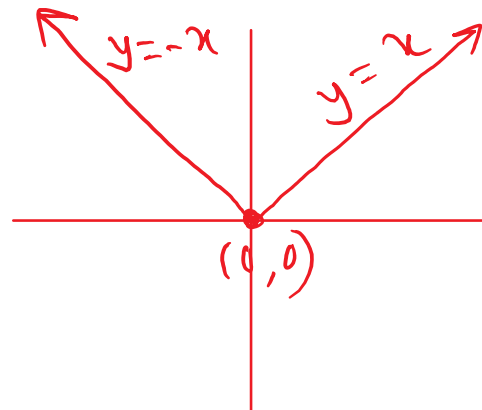
junction pt or vertex

 $y = -x$

$$f(x) = |x| = \begin{cases} x & ; x > 0 \\ 0 & ; x = 0 \\ -x & ; x < 0 \end{cases}$$

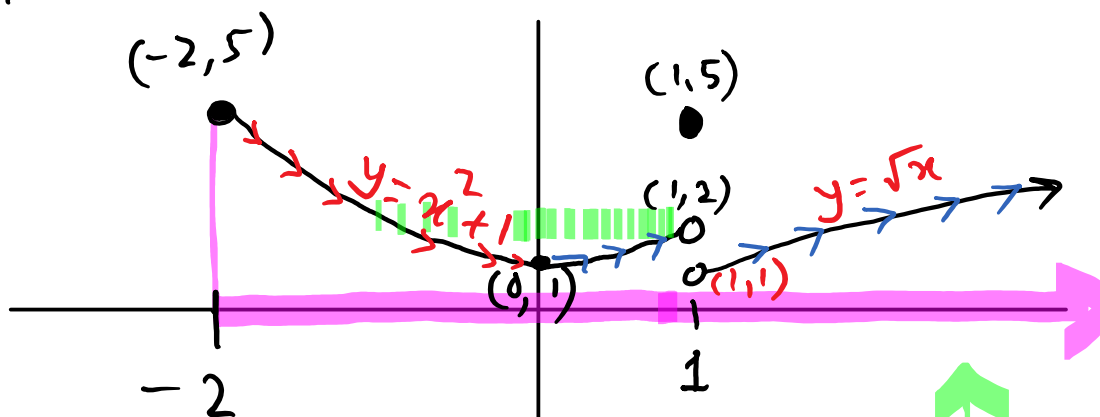
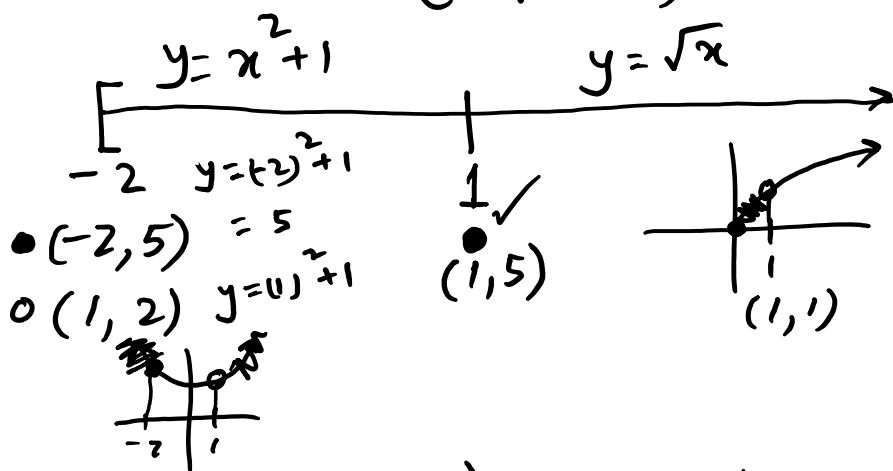
Annotations for the piecewise function:

- x ; $x > 0$: keep (yellow highlight)
- 0 ; $x = 0$: right of vertex (red oval)
- $-x$; $x < 0$: left of vertex (blue highlight)
- $-(x)$: left of vertex



ex2 Construct a piecewise function:

$$f(x) = \begin{cases} x^2 + 1; & -2 \leq x < 1 \\ 5; & x = 1 \\ \sqrt{x}; & x > 1 \end{cases}$$



① Domain: $[-2, \infty)$

Range: $[1, \infty)$

② Abs. max: $(-2, 5)$; None

Abs. min: $(0, 1)$

Local max: none

Local min: $(0, 1)$

③ Inc: $(0, 1) \cup (1, \infty)$

④ Where is the $f(x)$ "continuous"?
 $[-2, 1) \cup (1, \infty)$

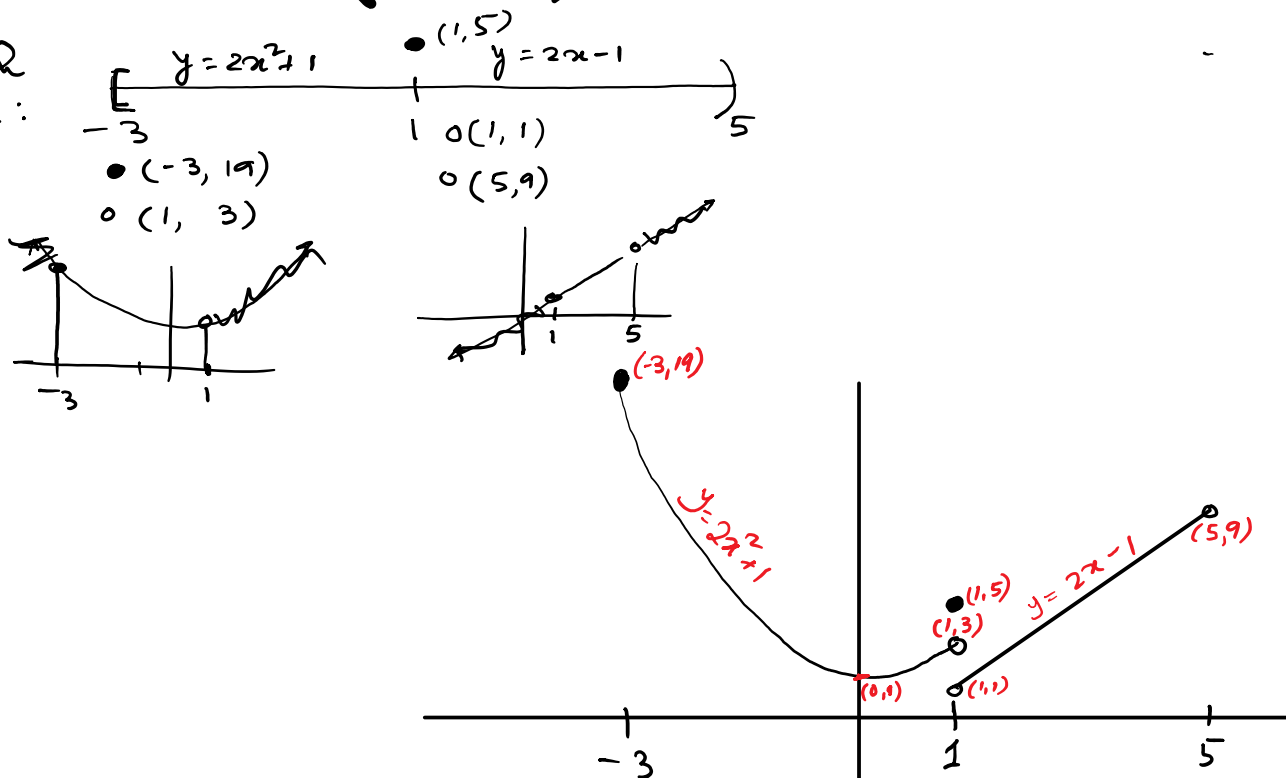
Dec: $(-2, 0)$

2.4p3(NEW)

Ex:

$$f(x) = \begin{cases} 2x^2 + 1 & ; -3 \leq x < 1 \\ 2x - 1 & ; 1 < x < 5 \\ 5 & ; x = 1 \end{cases}$$

Rough work:



questions:

a) Domain of $f(x)$? b) Range? $[1, 19]$
 $[-3, 5)$

c) Where is the function increasing?
 $(0, 1) \cup (1, 5)$

Decreasing? $(-3, 0)$

d) Where's the function continuous?
 (where is it unbroken?)
 $[-3, 1) \cup (1, 5)$

e) What are local extrema? local max: none
 local min: $(0, 1)$
 Abs. extrema: Abs. max: $(-3, 19)$
 Abs. min: $(0, 1)$

2.4#38(NEW)

38. Phone Charges The monthly cost C , in dollars, for calls from the United States to Japan on a certain phone plan is modeled by the function $C(x) = 0.24x + 5$, where x is the number of minutes used.

- What is the cost if you talk on the phone for $x = 50$ minutes?
- Suppose that your monthly bill is \$20.36. How many minutes did you use the phone?
- Suppose that you budget yourself \$40 per month for the phone. What is the maximum number of minutes that you can talk?
- What is the implied domain of C if there are 30 days in the month?
- Interpret the slope.
- Interpret the y-intercept.

Identify the variables:
 indep var x = Number of minutes/month called
 $C(x)$ = Total monthly cost for calls

a) $C(50) = \$0.24(50) + 5 = \boxed{\$17}$

b) If $C(x) = 20.36$, find x

$$\begin{array}{r} 20.36 = 0.24x + 5 \\ - 5 \qquad - 5 \\ \hline 15.36 = 0.24x \end{array}$$

$\boxed{64 \text{ mins} = x}$

d) Implied in this context
 $0 < x \leq 43,200$

$\downarrow \quad \downarrow \quad \downarrow$
 $(30)(24)(60)$
 days hrs mins

c) $C(x) \leq 40$

$$0.24x + 5 \leq 40$$

$$0.24x \leq 35$$

$$x \leq 145.83$$

or
 $\boxed{\text{No more than 145 mins}}$

e) slope means = $0.24 \uparrow$
 for each min $\rightarrow$
 You talk, the cost
 increases by 24 cents.

50. International Calling A cell phone company offers an international plan by charging \$30 for the first 80 minutes, plus \$0.50 for each minute over 80. *junction pt.*

(a) Write a linear model that relates the cost C , in dollars, of talking x minutes, assuming $x \geq 80$.

(b) What is the cost of talking 105 minutes? 120 minutes?

indep. var: $x = \text{Num. of minutes}$
 dep. var: $C(x) = \text{total charge (\$)}$

$$C(x) = \begin{cases} 30 & ; 0 < x \leq 80 \\ 30 + (x-80) \cdot 0.50 & ; x > 80 \end{cases}$$

$x = 70 \text{ min}$

$C(70) = \underline{30}$

$x = 85$

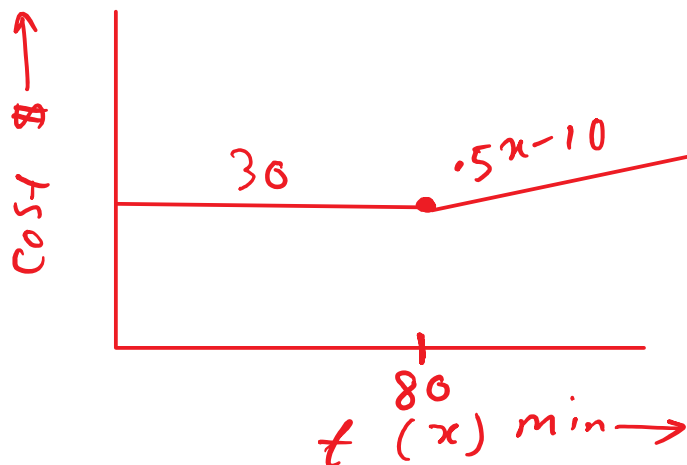
$C(85) = \$30 + (85 - 80)(0.50)$

$30 + 0.5(x-80)$
 $30 + 0.5x - 40$

$$C(x) = \begin{cases} 30; & 0 < x \leq 80 \\ 0.5x - 10; & x > 80 \end{cases}$$

$C(105) = 0.5(105) - 10 = \underline{\$42.50}$

$C(105) = 30 + (105 - 80) \cdot 0.5 = \42.50



52. Parking at O'Hare International Airport The short-term (no more than 24 hours) parking fee F (in dollars) for parking

x hours on a weekday at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 4 & \text{if } 0 < x \leq 3 \\ \frac{2 \operatorname{int}(x + 1) + 3}{35} & \text{if } 3 < x \leq 11 \\ 35 & \text{if } 11 < x \leq 24 \end{cases}$$

$2[x+1] + 3$
 $2[3.5+1] + 3$

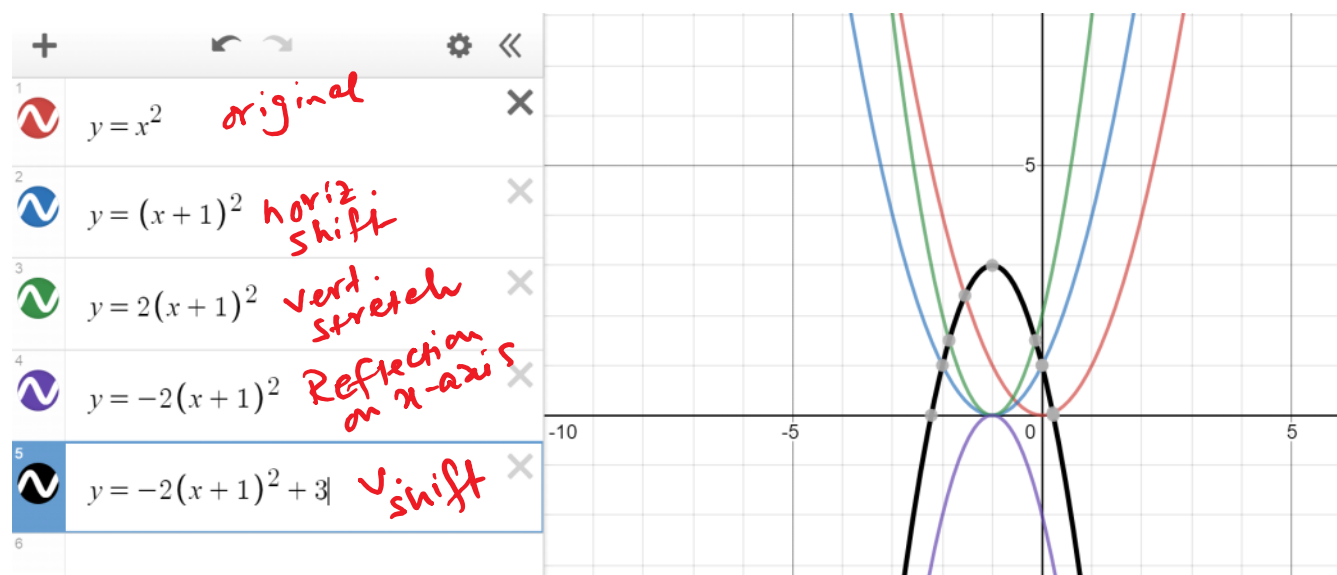
Determine the fee for parking in the short-term parking garage for

- (a) 2 hours (b) 7 hours
 (c) 15 hours (d) 8 hours and 24 minutes

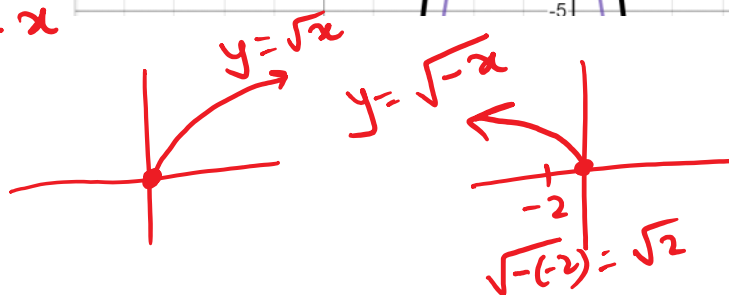
Source: O'Hare International Airport

$2[4.5] + 3$
 $2(4) + 3$
 $8 + 3$
 $= \$11$

2.5 Transformations

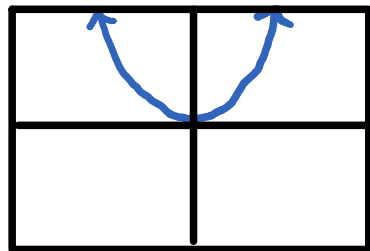


y-axis reflection
replace $x \rightarrow -x$



■ Introduction to Transformations of the graphs of functions
 you should try this on the calculator:
 start with a function from the
 "Library of Functions" say $y = x^2$

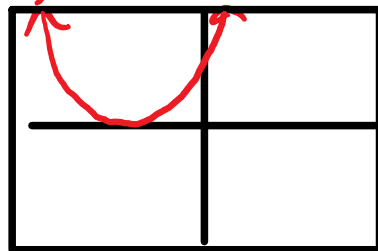
1. enter $y =$ $y_1 = x^2$
 notice vertex is $(0,0)$



2. $y_2 = (x+1)^2$

this is called a horizontal shift

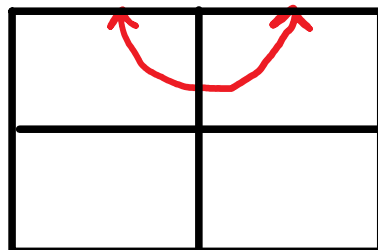
notice vertex shifted ←
 new vertex $(-1,0)$



3. Delete y_2 . Enter $y_2 = x^2 + 1$

this is called a vertical shift ↑

notice new vertex $(0,1)$

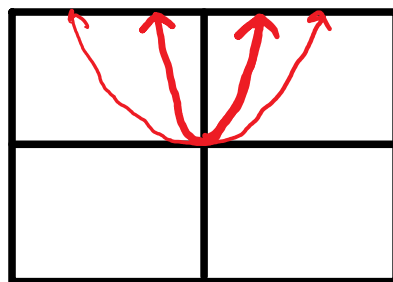


4. Delete y_2 . Enter $y_2 = 2x^2$

this is called a **vertical stretch**

notice vertex and x -intercepts stay same, "arms" are stretched.

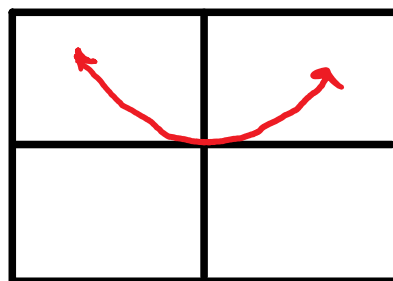
Every y -value = 2 (original y -value)



5. $y_2 = \frac{1}{2}x^2$ is a **horizontal stretch**

every y -value is $\frac{1}{2}$ of original y -value

vertical compression

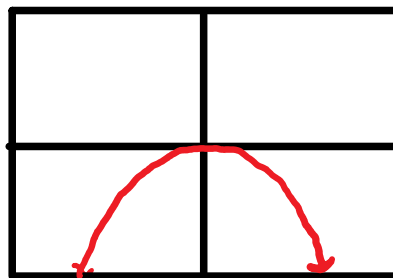


6. Delete y_2 . Enter $y_2 = -x^2$

all x -ints and vertex remains the same

Every y -value = opp. sign of original

Reflection on x -axis



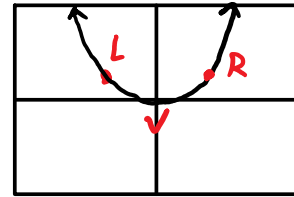
7. $y = x^2$ is already an even function, but if you want to **reflect on y -axis** then you Replace every x by $-x$

2.5 Transformation P2

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Vertical shifts		
$y = f(x) + k, \quad k > 0$	Raise the graph of f by k units.	Add k to $f(x)$.
$y = f(x) - k, \quad k > 0$	Lower the graph of f by k units.	Subtract k from $f(x)$.
Horizontal shifts		
$y = f(x + h), \quad h > 0$	Shift the graph of f to the left h units.	Replace x by $x + h$.
$y = f(x - h), \quad h > 0$	Shift the graph of f to the right h units.	Replace x by $x - h$.
Compressing or stretching		
$y = af(x), \quad a > 0$	Multiply each y -coordinate of $y = f(x)$ by a . Stretch the graph of f vertically if $a > 1$. Compress the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a .
$y = f(ax), \quad a > 0$	Multiply each x-coordinate of $y = f(x)$ by $\frac{1}{a}$. Stretch the graph of f horizontally if $0 < a < 1$. Compress the graph of f horizontally if $a > 1$.	Replace x by ax .
Reflection about the x-axis		
$y = -f(x)$	Reflect the graph of f about the x -axis.	Multiply $f(x)$ by -1 .
Reflection about the y-axis		
$y = f(-x)$	Reflect the graph of f about the y -axis.	Replace x by $-x$.

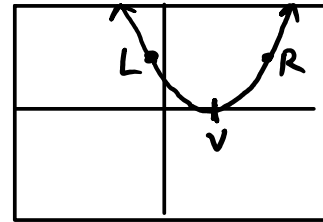
Frame-by-frame graphs of transformation.
 $y = -2(x - \frac{1}{2})^2 - 3$ [v. shift: do last]

① start (original : $y = x^2$ function)

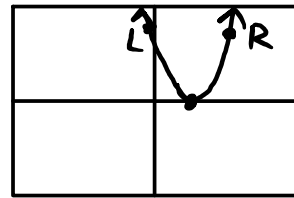


$V(0,0)$
 $L(-1,1)$
 $R(1,1)$

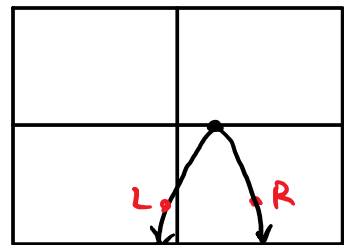
② $y = (x - \frac{1}{2})^2$
 $\frac{1}{2} \rightarrow$
 $V(\frac{1}{2}, 0)$
 $L(-\frac{1}{2}, 1)$
 $R(\frac{3}{2}, 1)$



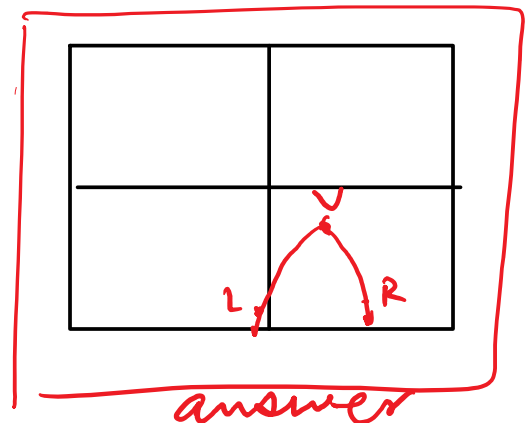
③ $y = 2(x - \frac{1}{2})^2$
 v. stretch
 $V(\frac{1}{2}, 0)$
 $L(-\frac{1}{2}, 2)$
 $R(\frac{3}{2}, 2)$



④ $y = -2(x - \frac{1}{2})^2$
 $V(\frac{1}{2}, 0)$
 $L(-\frac{1}{2}, -2)$
 $R(\frac{3}{2}, -2)$
 -3



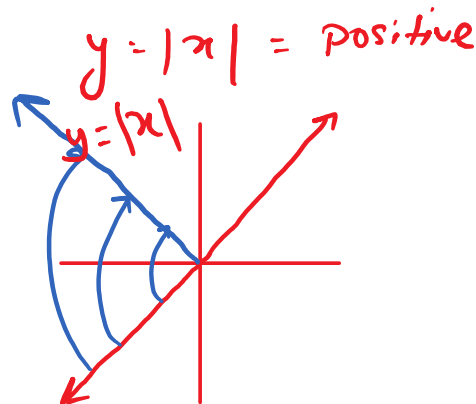
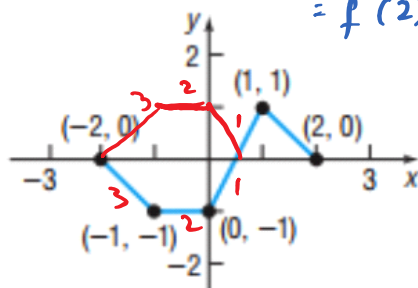
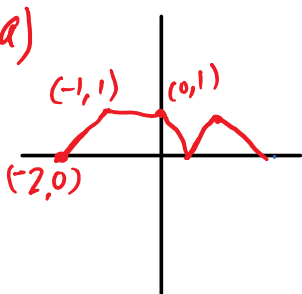
⑤ $y = -2(x - \frac{1}{2})^2 - 3$
 $V(\frac{1}{2}, -3)$
 $L(-\frac{1}{2}, -5)$
 $R(\frac{3}{2}, -5)$



78. The graph of a function f is illustrated in the figure.

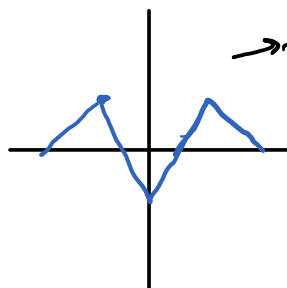
(a) Draw the graph of $y = |f(x)|$.

(b) Draw the graph of $y = f(|x|)$. $\rightarrow f(|-2|) = f(2) = 0$



b)

x	$ x $	$f(x)$
2	2	$f(2) = 0$
1	1	$f(1) = 1$
0	0	$f(0) = -1$
-1	1	$f(1) = 1$
-2	2	$f(2) = 0$



$\rightarrow$ the graph on the left of y-axis will be exactly the same as graph on right.

79. Suppose $(1, 3)$ is a point on the graph of $y = f(x)$.

(a) What point is on the graph of $y = f(x + 3) - 5$?

(b) What point is on the graph of $y = -2f(x - 2) + 1$?

(c) What point is on the graph of $y = f(2x + 3)$?

Start: $f(x)$

a) $f(x+3) - 5$
 $\leftarrow 3 \quad \downarrow$

$$\begin{array}{c} (1, 3) \\ -3 \quad -5 \\ \hline (-2, -2) \end{array}$$

$$f\left(2\left(x + \frac{3}{2}\right)\right)$$

$\rightarrow$ Shift input by $-\frac{3}{2}$

then mult. input by 2

b) $-2f(x-2) + 1$

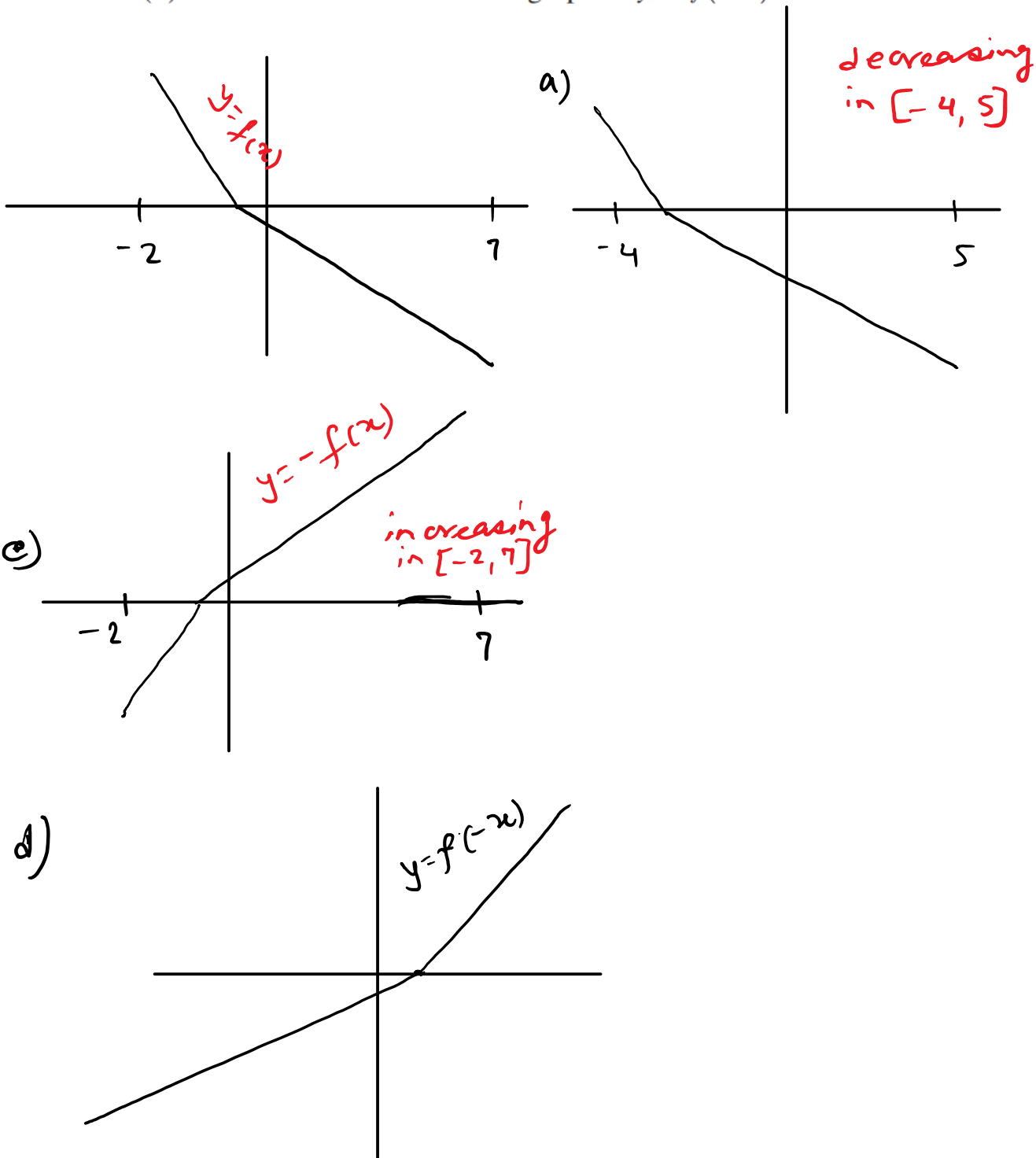
$\rightarrow 2$
 then $\times 2 \rightarrow$ output
 then flip
 then $+1$ output

$$\begin{array}{c} (1, 3) \\ +2 \\ \hline (3, 3) \\ \times 2 \\ \hline (3, 6) \\ \times -1 \\ \hline (3, -6) \\ +1 \\ \hline (3, -5) \end{array}$$

$$\begin{array}{c} (1, 3) \\ -3/2 \\ \hline (-1/2, 3) \\ \times 2 \\ \hline (-1, 3) \end{array}$$

38. Suppose that the function $y = f(x)$ is decreasing on the interval $[-2, 7]$.

- (a) Over what interval is the graph of $y = f(x + 2)$ decreasing?
- (b) Over what interval is the graph of $y = f(x - 5)$ decreasing?
- (c) What can be said about the graph of $y = -f(x)$?
- (d) What can be said about the graph of $y = f(-x)$?



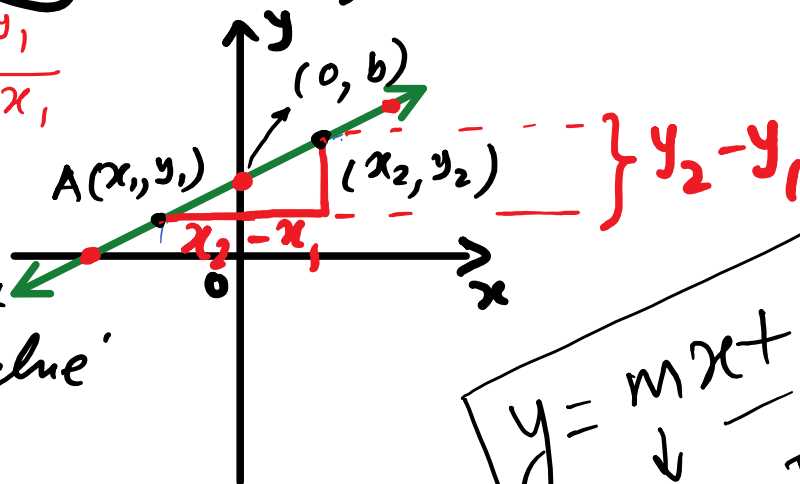
Straight Lines

Lines (straight lines)

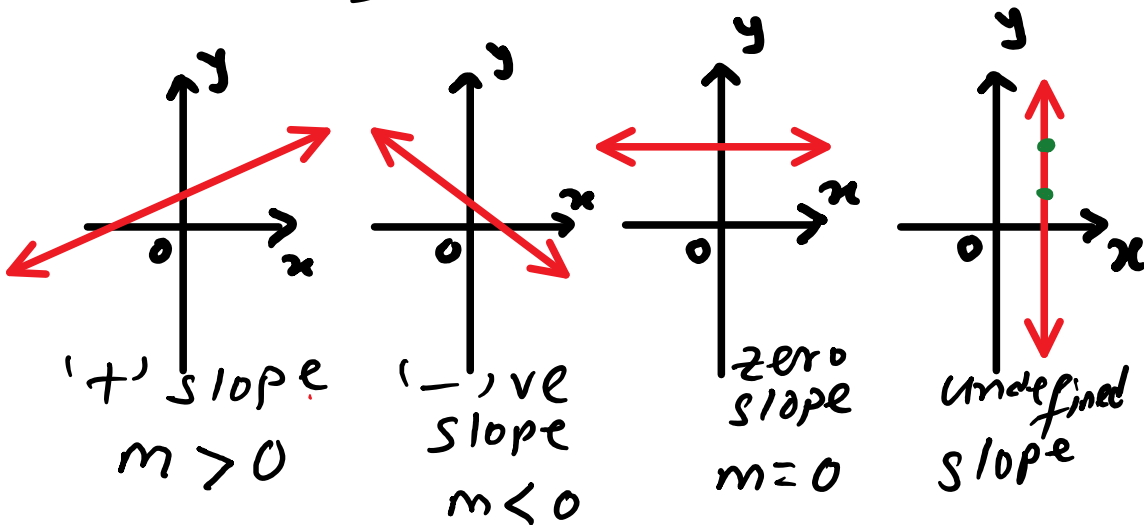
□ slope: $\frac{\text{Rise} \updownarrow}{\text{Run} \leftrightarrow} = \frac{y_2 - y_1}{x_2 - x_1}$

"means: The rate of change of y-value for 1 unit increase in x-value"

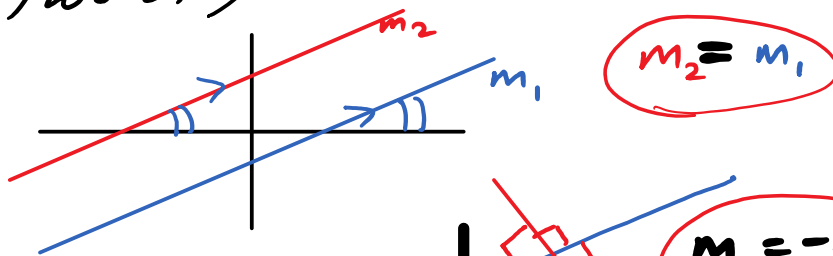
□ y-int: $(0, b)$



$y = mx + b$
 ↓
 Slope-Intercept form

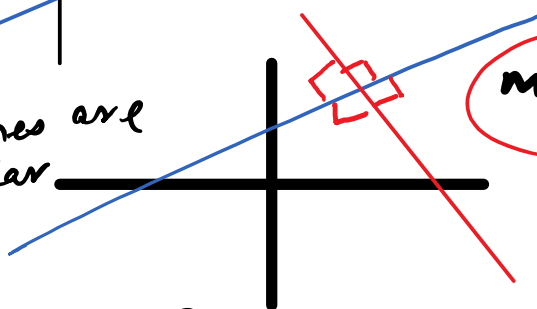


□ Two st. lines are ~~st~~ (parallel)



$m_2 = m_1$

□ Two st. lines are perpendicular



$m_1 = -\frac{1}{m_2}$

$m_1 = -\frac{2}{1}$
 $m_2 = \frac{1}{2}$

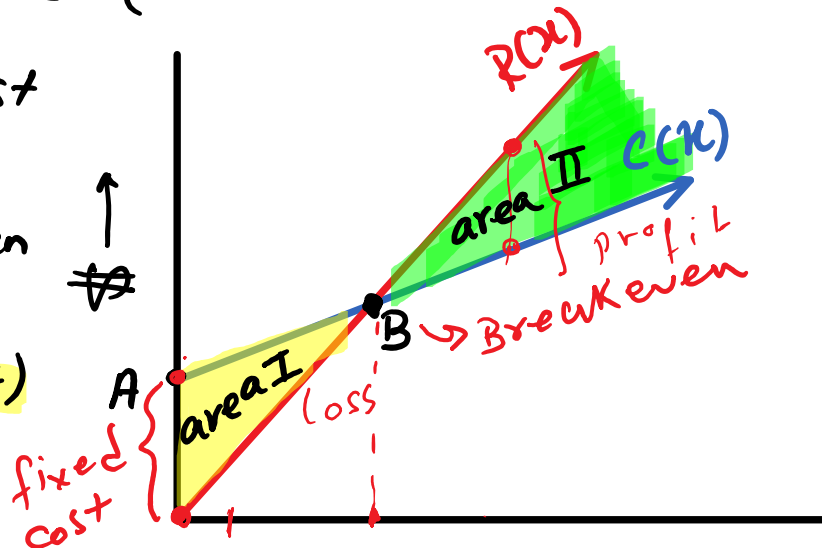
3.1. Common Linear models

Cost-Revenue (Break-even point)

→ A = fixed-cost value

→ B = Break-even point

$R(x) = C(x)$
at pt. B



→ Area I:

$C(x) > R(x)$: Loss

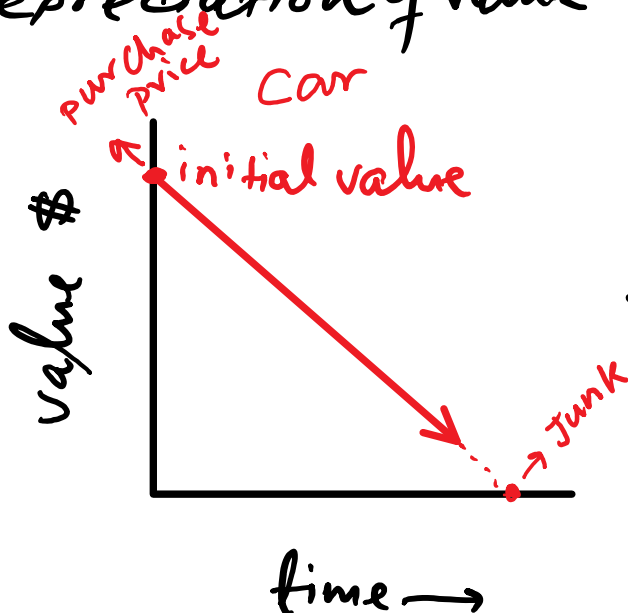
$x = \# \text{ of items produced or sold}$

→ Area II:

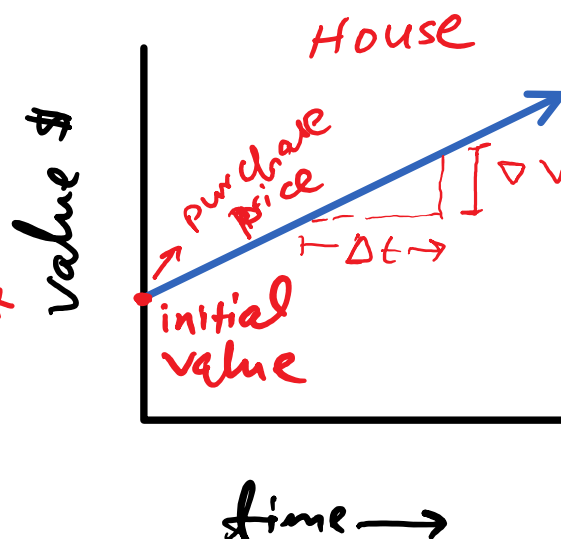
$R(x) > C(x)$: Profit

$$P(x) = R(x) - C(x)$$

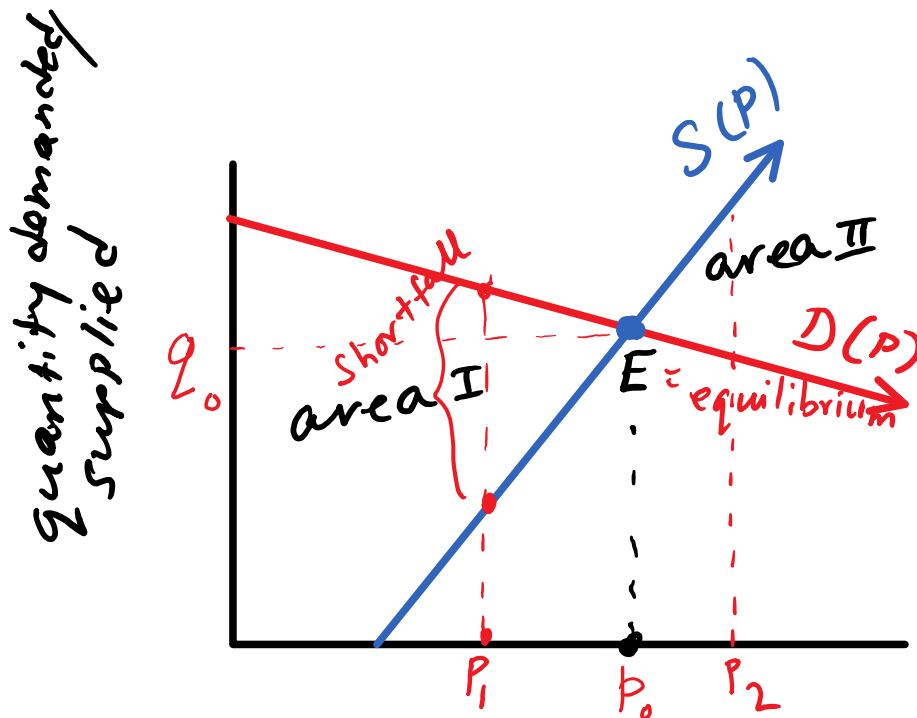
Depreciation of value



Appreciation of value



■ The Supply-Demand (Equilibrium point)



$p = \text{price/unit}$

→ $E = \text{Equilibrium point}$ $S(p) = D(p)$

→ Area I: $D(p) > S(p)$: shortage $D(p) - S(p)$

→ Area II: $S(p) > D(p)$: surplus $S(p) - D(p)$

→ $p_0 = \text{equilibrium price/unit}$

$q_0 = S(p_0) = D(p_0) = \text{equilibrium quantity}$

3.1#46(NEW)

46. Straight-line Depreciation Suppose that a company has just purchased a new machine for its manufacturing facility for \$120,000. The company chooses to depreciate the machine using the straight-line method over 10 years.

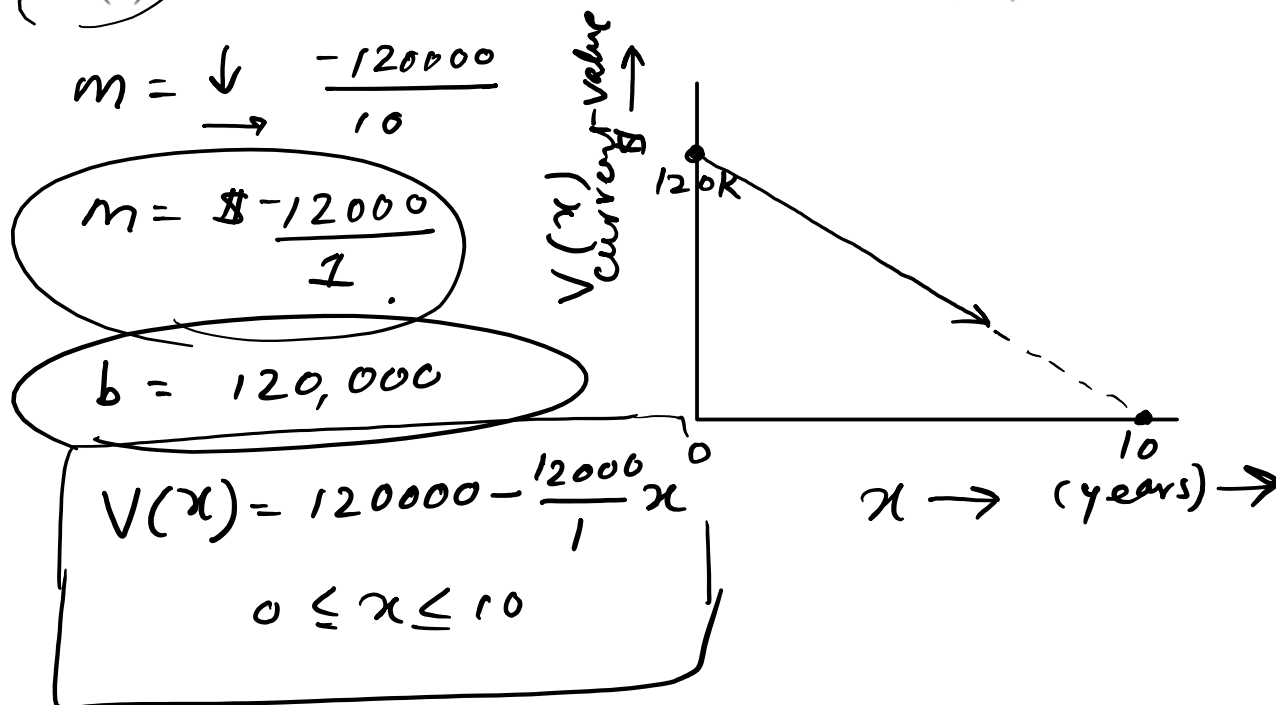
(a) Write a linear model that expresses the book value V of the machine as a function of its age x .

(b) What is the implied domain of the function found in part (a)?

(c) Graph the linear function.

(d) What is the book value of the machine after 4 years?

(e) When will the machine have a book value of \$72,000?



(d) $V(x) = 72000$
 $120000 - 12000x = 72000$

$x = 4 \text{ yrs}$

40. Supply and Demand Suppose that the quantity supplied S and the quantity demanded D of hot dogs at a baseball game are given by the following functions:

$$S(p) = -2000 + 3000p$$

$$D(p) = 10,000 - 1000p$$

where p is the price of a hot dog.

- Find the equilibrium price for hot dogs at the baseball game. What is the equilibrium quantity?
- Determine the prices for which quantity demanded is less than quantity supplied.
- What do you think will eventually happen to the price of hot dogs if quantity demanded is less than quantity supplied?

p = price / HT (indep)

$S(p)$ = Supply quantity } dep. var.

$D(p)$ = Demand quantity }

a) $S(p) = D(p) \rightarrow$ at equilibrium

$$\begin{aligned} -2000 + 3000p &= 10000 - 1000p \\ +2000 + 1000p &+ 2000 + 1000p \end{aligned}$$

$$4000p = 12000$$

$$p = 3$$

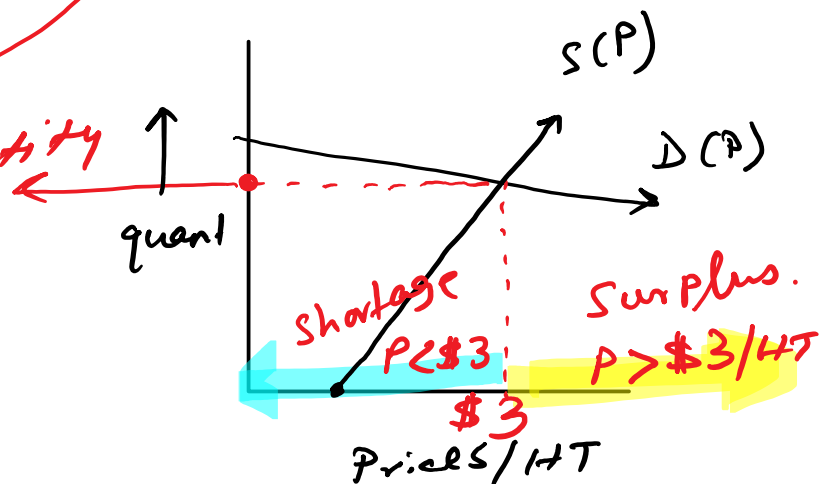
equilibrium quantity

$$S(3) = -2000 + 3000(3)$$

$$-2000 + 9000$$

7000 hot dogs

At equilibrium
Price / HT = \$3



\$12/item = selling price

$$44) R(x) = 12x$$

$$C(x) = 10x + 15000 \rightarrow \text{fixed cost} \quad \$10/\text{item} \rightarrow \text{cost/item}$$

$x \rightarrow$ Num. of items sold/produces

$C(x)$ } Total cost \$
 $R(x)$ } Total rev

Break-even sales:

$$R(x) = C(x) \quad 12x = 10x + 15000$$

$$2x = 15000$$

$x = 7,500$ items to be sold to break even.

$$R(7500) = \$$$

$$P(x) = R(x) - C(x) = 12x - (10x + 15000)$$
$$12x - 10x - 15000$$

$$P(x) = 2x - 15000$$

$$P(8000) = \$$$