Midterm

Please complete this midterm using RMarkdown. You may use your notes, our class videos, the textbook, etc. when working on the problems–the only thing you may not do is discuss the midterm with your classmates or others. If you have any questions, please email me.

Holiday Lights (18 points)

A town manager is beginning to prepare for the holiday season, and decides to decorate the town's shopping district with lights. However, the lights have not been used since the previous year, and they know from experience that some of the bulbs will have burned out. Each bulb will have to be inspected before the lights can be used. Assume that the probability that a single bulb has burnt out is $p = 0.06$, and further assume that all the bulbs are independent of each other.

  (a) How many bulbs should the manager expect to inspect before they find a burnt out bulb?

  (b) A string of lights has twenty bulbs. What is the probability that less than seven are burnt out?

  (c) What is the probability that exactly three of the twenty bulbs on a string of lights are burnt out?

  (d) What is the probability that the manager finds their first burnt out bulb on the tenth inspection?

  (e) What is the probability that the manager inspects at least twelve bulbs before they find one that is burnt out?

  (f) How many bulbs out of twenty on a string of lights should the manager expect to be burnt out?

Tree Sizes (7 pts)

Now the manager would like to make sure that they have enough trees to decorate the shopping district, and requests data on the sizes of trees at a local tree farm. Use the data below to answer the following questions from the town manager.

```
trees <- read.csv("https://vincentarelbundock.github.io/Rdatasets/csv/geepack/sitka89.csv")
```

  (a) Using `ggplot()`, create a histogram of `size`, the size of the trees at the farm as measured in $log(height * diameter^2)$. Your histogram should have 15 bins (3 pts).

  (b) Comment on the distribution of the tree sizes (4 pts).

Advertising (8 points)

In addition to decoration, the manager is responsible for coordinating the holiday preparedness of local businesses. They are specifically interested in analyzing the relationship between dollars spent on TV and radio advertising. Use the data below to answer the following questions from the manager.

```
advertising <- read.csv("https://vincentarelbundock.github.io/Rdatasets/csv/AER/DutchAdvert.csv")
```

  (a) Using `ggplot()`, create a scatterplot of `tv` vs. `radio` advertising dollars. Change the color of the points to a color of your choice (3 pts).

  (b) Comment on the relationship (4 pts).

  (c) Calculate the correlation between `tv` and `radio` advertising dollars (1 pt).

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Bookstore Coupons (15 points)

The manager knows that the owner of the local bookstore is planning to offer a coupon to encourage customers to shop at their store. The customers are expected to use the coupon at a rate of 3.5 per day.

(a) What is the probability that exactly five shoppers come in to use the coupon?

(b) What is the probability that no more than two customers come in to use the coupon?

(c) The coupon is valid for the entire month of December (31 days). What is the distribution of the average number of customers using the coupon?

(d) What is the probability that the average number of coupon users is between two and four?

(e) What is the probability that the average number of coupon users is at least three?

Bookstore Sales (15 points)

The bookstore owner knows that their daily sales are normally distributed with a mean of $478 and a standard deviation of $49 dollars.

(a) What is the probability that the daily sales are more than $521?

(b) The coupon the bookstore owner is planning to offer is a 10% off coupon–that is, customers will pay 90% of the total price. If the daily sales are represented by B, write an expression that represents the daily sales if all customers use the coupon.

(c) How much should the bookstore owner expect the daily sales to be with the coupon? What is the standard deviation of sales with the coupon?

(d) What is the probability that the daily sales are at least $400?

(e) What is the probability that the daily sales are at most $386?

Coffee Specials (15 points)

The coffeeshop next to the bookstore has also been offering a holiday special–a bakery treat (sugar cookie, gingerbread, panettone) and drink (latte, mocha, cappuccino) for $5. So far, they have sold the following:

	Latte	Mocha	cappuccino	Total
Sugar Cookie	75	50	25	150
Gingerbread	45	30	15	90
Panettone	24	16	8	48
Total	144	96	48	288

(a) What is the probability that a randomly chosen customer bought a latte (2 pts)?

(b) What is the probability that a randomly chosen customer bought gingerbread (2 pts)?

(c) What is the probability that a randomly chosen customer bought a mocha and panettone (2 pts)?

(d) What is the probability that a randomly chosen customer bought a cappuccino, given that they bought a sugar cookie (2 pts)?

(e) What is the probability that a randomly chosen customer bought panettone, given that they bought a cappuccino (2 pts)?

(f) Are lattes and gingerbread independent? Give a mathematical justification (3 pts).

Stationary Sale (8 points)

The stationary store across the street from the coffee shop is planning a sale on holiday cards. The first box of cards is $7.99, but customers can buy up to three additional boxes for $5.99 each. They know approximately how many customers will buy one, two, three, or four boxes; however, some customers come in and do not buy anything. Here is a distribution of their expected sales, S:

	$0	$7.99	$13.98	$19.97	$25.96
$P(S)$	0.18	0.16	0.26	0.26	$P(\$25.96)$

(a) Find the value of $P(\$25.96)$ that will make this distribution a valid distribution (2 pts).

(b) What is $E(S)$ (2 pts)?

(c) What is $Var(S)$ (3 pts)?

(d) What is the probability a customer buys at least one box of stationary (1 pt)?

Customer Plans (14 points)

Finally, the town manager polls residents and shoppers to plan ahead for the holiday rush. Of a random sample of 324, he finds that 191 are planning on doing at least some of their holiday shopping in the town's shopping district.

(a) Build a 90% confidence interval for the true proportion of shoppers who are planning on doing at least some of their holiday shopping in the town's shopping district (9 pts).

(b) What is the margin of error of the confidence interval (3 pts)?

(c) If the manager increases the confidence level, will the interval become wider or narrower (2 points)?