

$$1. \int 3 \sin^2 x \, dx$$

$$3 \int \sin^2 x \, dx$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$3 \int \left(\frac{1 - \cos(2x)}{2} \right) dx$$

$$3 \left(\int \frac{1}{2} dx - \int \frac{\cos(2x)}{2} dx \right)$$

$$\frac{3}{2} \int dx - \frac{3}{2} \int \cos(2x) dx$$

$$\frac{3}{2} \int dx = \frac{3}{2} x$$

$$\frac{3}{2} \int \cos(2x) dx$$

$$\text{Let } u = 2x \quad du = 2$$

$$\frac{3}{4} \int \cos u \, du = \frac{3}{4} \sin u = \frac{3}{4} \sin(2x)$$

$$\int 3 \sin^2 x \, dx = \frac{3}{2} x - \frac{3}{4} \sin(2x) + C$$

$$2. \frac{dy}{dt} = \frac{8t^2}{3y}$$

$$3y \, dy = 8t^2 \, dt$$

$$\int 3y \, dy = \int 8t^2 \, dt$$

$$\frac{3y^2}{2} = \frac{8t^3}{3} + C$$

$$y^2 = \frac{2}{3} \left(\frac{8t^3}{3} + C \right)$$

$$y^2 = \frac{16t^3}{9} + C$$

$$y = \pm \sqrt{\frac{16t^3}{9} + C}$$

$$3. \int e^{9x} \cos(8x) dx$$

$$u = \cos(8x)$$

$$dv = e^{9x}$$

$$du = -8 \sin(8x) dx$$

$$v = \frac{e^{9x}}{9}$$

$$uv - \int v du$$

$$\frac{e^{9x} \cos(8x)}{9} + \int \frac{8}{9} e^{9x} \sin(8x) dx$$

$$u = \sin(8x)$$

$$dv = e^{9x}$$

$$du = 8 \cos(8x)$$

$$v = \frac{e^{9x}}{9}$$

$$= \frac{e^{9x} \cos(8x)}{9} + \frac{8}{9} \left(\frac{1}{9} e^{9x} \sin(8x) - \frac{8}{9} \int e^{9x} \cos(8x) dx \right)$$

$$= \frac{e^{9x} \cos(8x)}{9} + \frac{8}{81} e^{9x} \sin(8x) - \frac{64}{81} \int e^{9x} \cos(8x) dx$$

$$\int e^{9x} \cos(8x) dx + \frac{64}{81} \int e^{9x} \cos(8x) dx = \frac{e^{9x} \cos(8x)}{9} + \frac{8}{81} e^{9x} \sin(8x)$$

$$\frac{145}{81} \int e^{9x} \cos(8x) dx = \frac{e^{9x} \cos(8x)}{9} + \frac{8}{81} e^{9x} \sin(8x)$$

$$\int e^{9x} \cos(8x) dx = \frac{81}{145} \left(\frac{e^{9x} \cos(8x)}{9} + \frac{8}{81} e^{9x} \sin(8x) \right)$$

$$= \frac{9 e^{9x} \cos(8x)}{145} + \frac{8 e^{9x} \sin(8x)}{145}$$

$$4 \int_4^{12} x^3 dx \quad n=1$$

$$h = \frac{b-a}{n} = \frac{12-4}{1} = 8$$

$$f\left(\frac{x_0 + x_1}{2}\right) = f\left(\frac{4+12}{2}\right) = f(8) = 8^3 = 512$$

$$M(1) = 8 \times 512 = 4096$$

$$n=2$$

$$h = \frac{12-4}{2} = 4$$

$$f\left(\frac{x_0 + x_1}{2}\right) = f\left(\frac{4+8}{2}\right) = f(6) = 6^3 = 216$$

$$f\left(\frac{x_1 + x_2}{2}\right) = f\left(\frac{8+12}{2}\right) = f(10) = 10^3 = 1000$$

$$M(2) = 4(216 + 1000) = 4864$$

$$n=4$$

$$h = \frac{12-4}{4} = 2$$

$$f\left(\frac{x_0 + x_1}{2}\right) = f\left(\frac{4+6}{2}\right) = f(5) = 125$$

$$f\left(\frac{x_1 + x_2}{2}\right) = f\left(\frac{6+8}{2}\right) = f(7) = 343$$

$$f\left(\frac{x_2 + x_3}{2}\right) = f\left(\frac{8+10}{2}\right) = f(9) = 729$$

$$f\left(\frac{x_3 + x_4}{2}\right) = f\left(\frac{10+12}{2}\right) = f(11) = 1331$$

$$M(4) = 2(125 + 343 + 729 + 1331)$$

$$= 5056$$

5

$$\int \frac{dx}{x\sqrt{25-x^{10}}}$$

$$u = \sqrt{25-x^{10}}$$

$$\frac{du}{dx} = -\frac{5x^9}{\sqrt{25-x^{10}}}$$

$$= \frac{1}{5} \int \frac{1}{u^2-25} du$$

$$\frac{1}{5} \int \frac{1}{(u-5)(u+5)} du$$

$$\frac{1}{(u-5)(u+5)} = \frac{A}{u-5} + \frac{B}{u+5}$$

$$1 = A(u+5) + B(u-5)$$

$$A+B=0$$

$$A=-B$$

$$5A-5B=1$$

$$-5B-5B=1$$

$$-10B = \frac{1}{10}$$

Ans

$$B = -\frac{1}{10}$$

$$A = \frac{1}{10}$$

$$= \frac{1}{5} \int \left(\frac{1}{10(u-5)} - \frac{1}{10(u+5)} \right) du$$

$$\frac{1}{10} \int \frac{1}{u-5} du - \frac{1}{10} \int \frac{1}{u+5} du$$

$$= \frac{\ln|u-5|}{10} - \frac{\ln|u+5|}{10} + C$$

$$= \frac{\ln|\sqrt{25-x^{10}}-5|}{50} - \frac{\ln|\sqrt{25-x^{10}}+5|}{50} + C$$

$$6. \int_0^{\pi/6} 2^{\cos 3t} \sin(3t) dt$$

$$u = \cos(3t) \quad du = -3 \sin(3t) dt$$

$$\frac{du}{dt} = -3 \sin(3t)$$

$$dt = -\frac{1}{3 \sin(3t)} du$$

$$= -\frac{1}{3} \int 2^u du$$

$$= -\frac{1}{3} \left(\frac{2^u}{\ln(2)} \right) = -\frac{1}{3} \frac{2^{\cos(3t)}}{\ln(2)} \Big|_0^{\pi/6}$$

$$= \frac{-2}{3 \ln(2)} +$$

$$= \frac{1}{3 \ln(2)}$$

7 (a) elevation, 25000 ft

feet

$$\frac{1}{3} = e^{-25000K}$$

$$K = \frac{\ln(1/3)}{-25000} = 0.0000439$$

half pressure

$$\frac{1}{2} = e^{-0.0000439t}$$

$$t = \frac{\ln(1/2)}{-0.0000439} = 15773 \text{ ft}$$

1% pressure

$$0.01 = e^{-0.0000439t}$$

$$t = \frac{\ln(0.01)}{-0.0000439} = 104795 \text{ ft}$$

$$8 \int_0^{\pi/2} \tan(13x) dx$$

$$= \int_0^{\pi/2} \frac{\sin(13x)}{\cos(13x)} dx$$

$$u = \cos(13x)$$

$$du = -13 \sin(13x) dx$$

$$\int_0^{\pi/2} -\frac{1}{13u} du = -\frac{1}{13} \ln|u|$$

$$= -\frac{1}{13} \ln|\cos(13x)| \Big|_0^{\pi/2}$$

$$= -\infty + 0 = \infty$$

diverges

$$9. \int \frac{14x^3 - 28x^2 + 8}{x^2 - 2x} dx$$

$$x^2 - 2x \overline{) \begin{array}{r} 14x^3 - 28x^2 + 8 \\ 14x^3 - 28x^2 \\ \hline 8 \end{array}} \quad 14x + \frac{8}{x^2 - 2x}$$

$$\int \left(14x + \frac{8}{x^2 - 2x} \right) dx$$

$$\frac{14x^2}{2} + 8 \int \frac{1}{x^2 - 2x} dx$$

$$8 \int \frac{1}{(x-1)^2 - 1} dx$$

$$\text{Let } u = x - 1$$

$$du = dx$$

$$\begin{aligned}
 &= 8 \int \frac{1}{u^2-1} du = 8 \left(-\frac{\ln|u+1|}{2} - \frac{\ln|u-1|}{2} \right) \\
 &\quad 8 \left(- \left(\frac{\ln|x-1+1|}{2} - \frac{\ln|x-1-1|}{2} \right) \right) \\
 &= -8 \left(\frac{1}{2} \ln|x| - \frac{1}{2} \ln|x-2| \right)
 \end{aligned}$$

$$= 7x^2 - 4\ln|x| + 4\ln|x-2| + C$$

$$10. \quad \frac{x}{y} = \frac{0.5}{4} = \frac{1}{8}$$

$$x = \frac{y}{8}$$

(a)

$$A(y) = \pi \left(\frac{y}{8} \right)^2 = \frac{\pi y^2}{64}$$

$$\int_0^h \rho g A(y) (h-y) dy$$

$$\int_0^4 1000 \times 9.8 \times \frac{\pi y^2}{64} (4-y) dy$$

$$\int_0^4 \frac{1225\pi}{8} y^2 (4-y) dy$$

$$\frac{1225\pi}{8} \int_0^4 (4y^2 - y^3) dy$$

$$\frac{1225\pi}{8} \left(\frac{4}{3} y^3 - \frac{y^4}{4} \right) \Big|_0^4 = \frac{9800\pi}{3} \text{ J}$$

$$(b) \quad \frac{1225\pi}{8} \left(\frac{4}{3} y^3 - \frac{y^4}{4} \right) \Big|_0^2 = \frac{6125\pi}{6} \text{ J}$$

$$11 \quad x = 2 \tan^{-1} \theta$$

$$dx = \frac{2}{\theta^2 + 1} d\theta$$

$$= \int \frac{\frac{2}{\theta^2 + 1}}{1 + \sin(2 \tan^{-1} \theta) + \cos(2 \tan^{-1} \theta)} d\theta$$

$$= \int \frac{2}{(1 + \theta^2) (1 + (2 \sin(\tan^{-1} \theta) \cos(\tan^{-1} \theta)) + \cos^2(\tan^{-1} \theta)) - \sin^2(\tan^{-1} \theta)}$$

$$\theta = \tan\left(\frac{x}{2}\right) \quad \sin(\tan^{-1} \theta) = \frac{\theta}{\sqrt{1 + \theta^2}}$$

$$\int \frac{dx}{1 + \sin x + \cos x} = \int \frac{d\theta}{\theta + 1}$$

$$= \ln |\theta + 1| + C$$

$$= \ln \left| \tan\left(\frac{x}{2}\right) + 1 \right| + C$$

$$12 \quad \int \frac{18}{(9x+1)^3} dx$$

$$u = 9x + 1 \quad du = 9 dx$$

$$18 \int \frac{1}{9u^3} du = 2 \int \frac{1}{u^3} du = 2 \frac{u^{-2}}{-2} + C$$

$$= -(9x+1)^{-2} + C$$

$$= \frac{-1}{(9x+1)^2} + C$$

$$13 \quad y = x^3 - 8$$

$$x = 0 \Rightarrow y = 0 - 8 = -8$$

$$y = 0 \Rightarrow 0 = x^3 - 8 \Rightarrow x = 2$$

$$(a) \int_0^2 \pi(y)^2 dx = \int_0^2 \pi(x^3 - 8)^2 dx = \pi \int_0^2 (x^6 - 16x^3 + 64) dx$$

$$\pi \left(\frac{x^7}{7} - 4x^4 + 64x \right) \Big|_0^2 = \frac{576\pi}{7}$$

$$(b) \int_0^2 -2\pi x (x^3 - 8) dx$$

$$= -2\pi \int_0^2 (x^4 - 8x) dx$$

$$= -2\pi \left(\frac{x^5}{5} - 4x \right) \Big|_0^2 = \frac{96\pi}{5}$$

The volume about the ~~ax~~ x-axis is greater.

$$14 \quad \int_0^2 (2t + 1) dt = t^2 + t \Big|_0^2 = 4 + 2 = \frac{6}{2} = 3$$

$$15 \quad x^2 = y^4 (1 - y^3)$$

$$x = y^2 \sqrt{1 - y^3}$$

$$= \int_0^1 y^2 \sqrt{1 - y^3} dy$$

$$\text{Let } t = 1 - y^3 \quad dt = -3y^2 dy$$

$$= \int_0^1 \frac{\sqrt{t}}{3} dt = -\frac{1}{3} \left(\frac{2}{3} t^{3/2} \right) \Big|_0^1 = \frac{2}{9}$$

$$\text{Area} = \frac{2}{9} \times 2 = \frac{4}{9}$$