

Mini-Application Project

Falling Bodies

Overview

Reference: Watch the video <https://www.youtube.com/watch?v=TMQdGgQuY5w>

Sir Isaac Newton stated [three basic laws of motion](#). The second law states that force (F) is equal to mass (m) times the acceleration (a), that is, $F=ma$. Since the acceleration is the second derivative of the position, we can use this to write second order differential equations on the motion of an object. In some problems, we can note that acceleration is the first derivative of the velocity and write an equation of motion for the velocity that is a first order equation.

Problems

Consider the problem of dropping an object from a high bridge. We'll consider two problems: (1) no air resistance on the falling body, and (2) the effect of air resistance drag on the object.

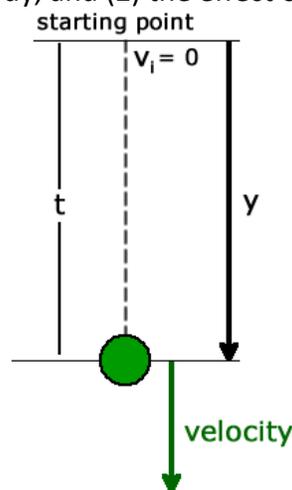


Figure 1 – Falling body – dropping an object from a bridge.

1. (4 pts) Write and solve a differential equation for the falling body without air resistance (that is, no drag). Note that the only force acting on the body is its weight due to gravity, that is, $W=mg$ where g is the acceleration due to (Earth's) gravity and is equal to -32 feet/second² or -9.8 meters per second squared. Derive equations for both the position and velocity. Note that the motion becomes independent of the mass (as demonstrated by Galileo at the Leaning Tower of Pisa). For specifics, use that the height of the bridge is 500 feet. (For the coordinate system, take the height y as positive in the downward direction with the bridge at $y=0$.)

2. (6 pts) Note that the gravitational acceleration constant g acts as a “parameter” for this problem. That is, it is constant for the context of a problem on Earth, but would change from planet to planet (that is, your weight is different on different planets.) Solve the same equations for both the moon and Mars. Can you develop a “closed form” solution in terms of this parameter? (That is, a solution in which you can plug in your value of the gravitational acceleration and get the correct equations for the position and velocity without resolving the differential equation.)
3. (7 pts) Now solve the same problem but with air resistance. The air resistance is modeled as a drag term whose force is proportional to the velocity (see the video or similar references.) Use that the acceleration is the first derivative of the velocity to write and solve a first order equation (again, see the reference). As in the video, use that the drag constant is $k=0.125$.
4. (8 pts) Note that the equations now have three parameters – the mass m , the gravitational acceleration g , and the drag coefficient k . Examine the effects of these parameters on the solution. See if you can find a closed form solution in terms of these parameters. Graph the position and velocity for example values of the parameters. What do you notice about the velocity? As a test case, find suitable values of these parameters for Mars and compare the solution with that of Earth.