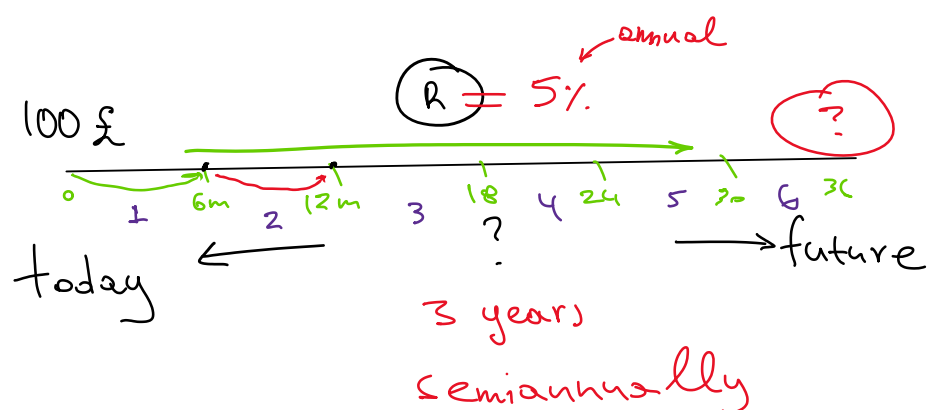


Monday

Monday, 15 March 2021

14:48

Compounding (Present $\rightarrow$ Future) : ($\times$) multiplication
 Discounting (Present $\leftarrow$ Future) : ($\div$) division $\left[\frac{1}{x} = (x)^{-1} \right]$



Compounding ($\times$)

$$6m: 100 \left(1 + \frac{5\%}{2} \right)$$

$$12m: 100 \left(1 + \frac{5\%}{2} \right) \times \left(1 + \frac{5\%}{2} \right) = 100 \left(1 + \frac{5\%}{2} \right)^2 \neq 100 (1 + 5\%)$$

Nikita: 100 £ for 1 year @ 5% annual $\rightarrow$ 105 £

Nikos: 100 £ for 6 months @ 5% annual $\rightarrow$...

more frequent

continuous compounding

$$6m: 102.5$$

$$6m: 102.5 \text{ for 6 months @ } 5\% \text{ annual } 102.5 \cdot \left(1 + \frac{0.025}{1} \right) = 105.0625$$

$$100 \cdot e^{rt} \leftarrow \text{years}$$

$$= 100 \cdot e^{0.05 \cdot 1} = 105.127$$

number of compounding per year

$\rightarrow$ the higher

$$36m: 100 \left(1 + \frac{R}{n} \right)^{n \cdot y} = 115.96$$

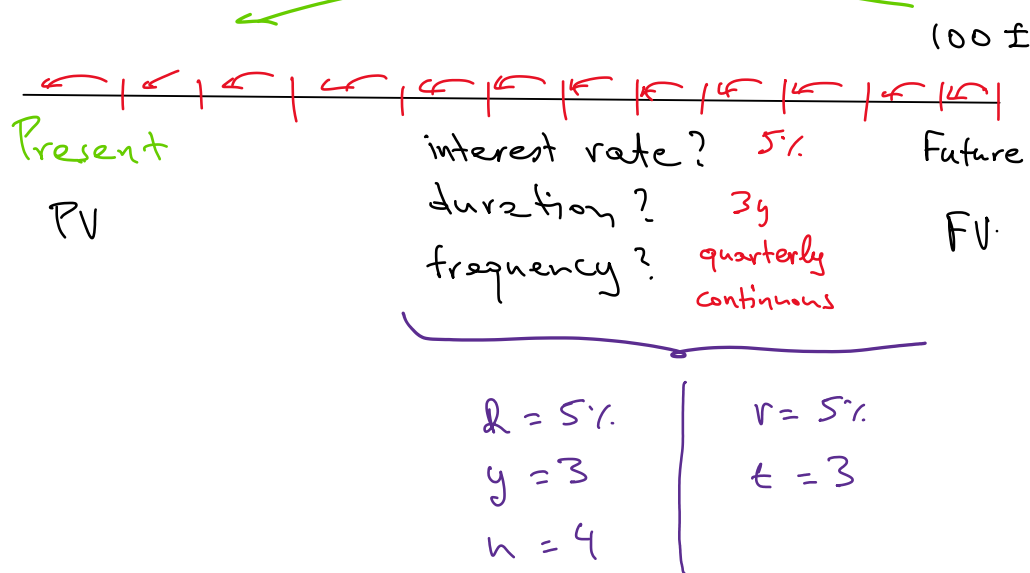
$$C_0 \left(1 + \frac{R}{n} \right)^{n \cdot y} \quad \left(\begin{array}{l} \sim n=2 \\ \sim y=3 \end{array} \right)$$

$$C_0 \cdot e^{rt} \leftarrow \text{short rate}$$

Discounting

$$WACC \approx \frac{X_1}{X} \cdot r_1 + \frac{X_2}{X} \cdot r_2 + \frac{X_3}{X} \cdot r_3$$

$$X = X_1 + X_2 + X_3$$



$$C_0 = C_F \cdot \left(1 + \frac{r}{n} \right)^{-n \cdot y}$$

$$C_0 = \frac{C_F}{\left(1 + \frac{r}{n} \right)^{n \cdot y}} = C_F \cdot \left(1 + \frac{r}{n} \right)^{-n \cdot y}$$

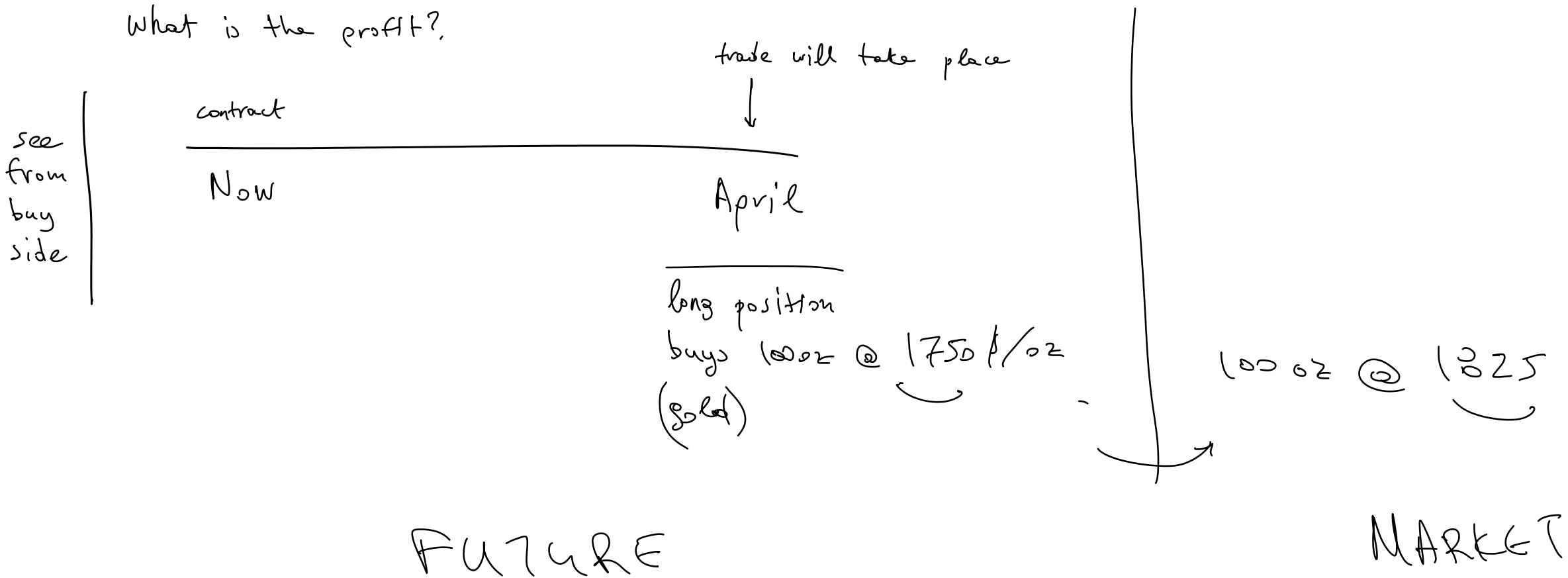
$$C_0 = C_F \cdot e^{-rt}$$

$$C_0 = 100 \cdot \left(1 + \frac{0.05}{4} \right)^{-4 \cdot 3} = 86.15$$

$$C_0 = 100 \cdot e^{-0.05 \cdot 3} = 86.07$$

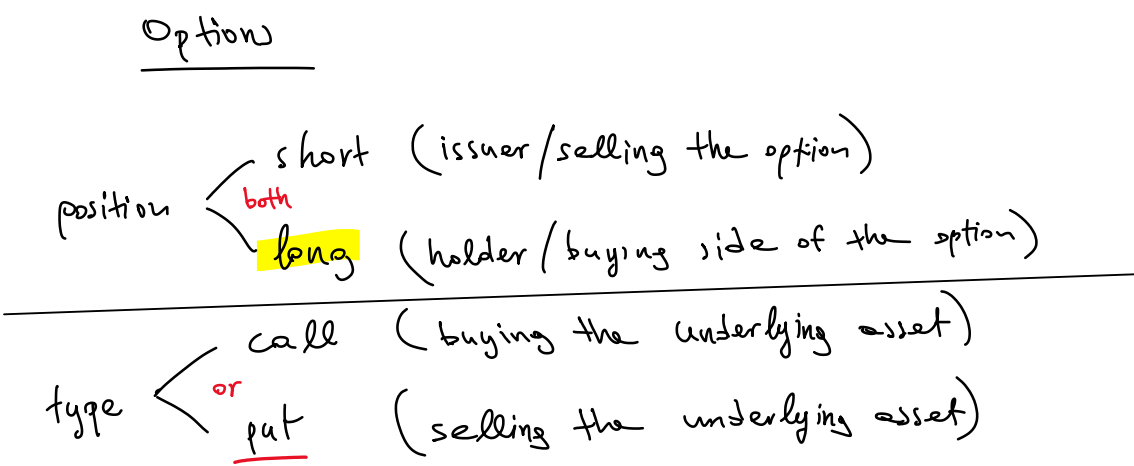
$\leftarrow$ the lowest

Slide 15: long future to buy 100oz of gold @ 1750\$/oz in April
April → spot price 1825\$/oz



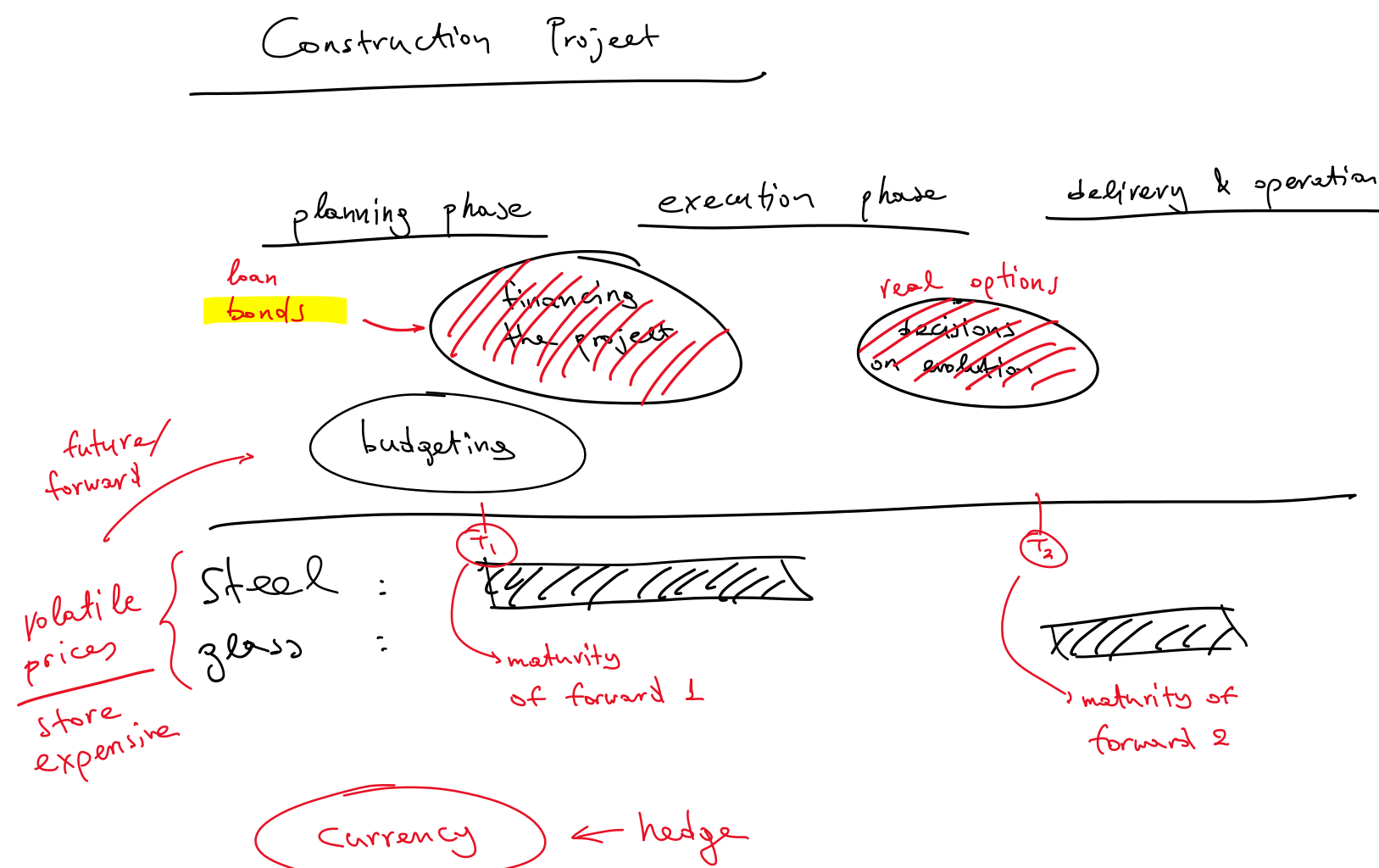
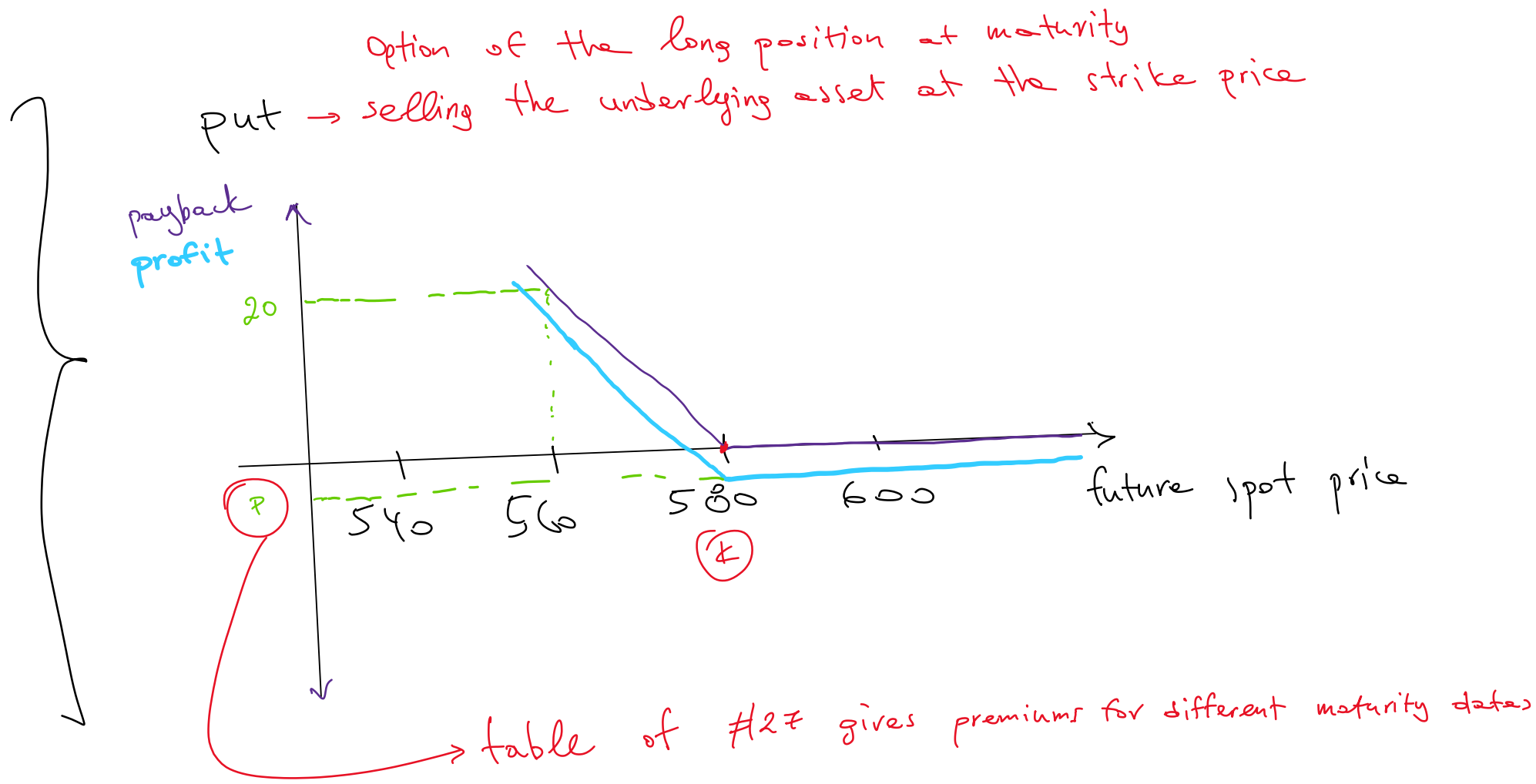
• cheaper (+) → profit
• price difference → profit (market - future)

$(1825 - 1750) \cdot 100 = 7500$



Strike price K (price of underlying asset)
Maturity date T American (Till when can be exercised)
European (When can be exercised)
Premium (price of option)

slide 27: Stock google, spot 561.32\$/561.51\$
Strike price 580\$ (put option)
Maturity T
European



Tuesday1

Tuesday, 16 March 2021

15:06

Compounding

Discounting

Discrete

$$F = A \left(1 + \frac{R}{n}\right)^{n \cdot y}$$

$$F = A \left(1 + \frac{R}{n}\right)^{-n \cdot y}$$

Continuous

$$F = A \cdot e^{rt}$$

$$F = A \cdot e^{-rt}$$

F: Future Value

A: Principal

R: Nominal IR / r: short rate

n: Frequency

y: Number years (discrete)

t: time (continuous)

Bond < Companies
governments

B_t: Bond price

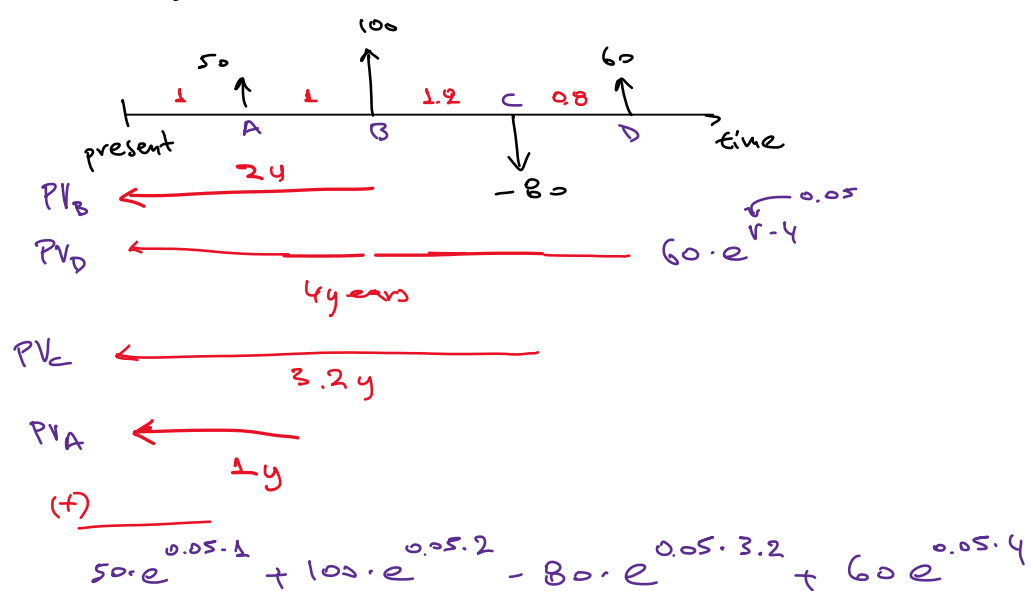
B₀: Nominal value

C: Coupon (annual)

R: Interest rate

y: years

Cash flows



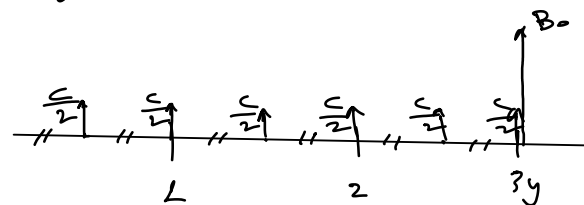
$$F = d \frac{\left(1 + \frac{R}{n}\right)^{n \cdot y} - 1}{R/n}$$

Semi-annual payments } $n=2$
 $y = \text{number of years}$

$$A = d \frac{1 - \left(1 + \frac{R}{n}\right)^{-n \cdot y}}{R/n} = d \cdot \left(1 + \frac{R}{n}\right)^{-1} + d \cdot \left(1 + \frac{R}{n}\right)^{-2} + \dots$$

100 £ nominal value

3 years duration, Semiannual coupon (C=5£)



$$B_t = \underbrace{\frac{C}{n} \frac{1 - \left(1 + \frac{R}{n}\right)^{-n \cdot y}}{R/n}}_{\text{coupons discounted}} + \underbrace{B_0 \left(1 + \frac{R}{n}\right)^{-n \cdot y}}_{\text{discounted nominal}}$$

Premium Bond

Discount Bond

Par Bond

$R = R_{YTM}$? → return if bond is purchased @ B_t
and withheld till maturity

NPV ~ IRR
Fix solve

Tuesday 2

Tuesday, 16 March 2021 17:08

S : Spot price today

F : Future/Forward price today

T : Time until delivery

r : Risk-free interest rate
 $\{r - r_f\}$

$$F = S \cdot e^{rT} \quad \leftarrow \text{price of future}$$

$$F = (S - I) \cdot e^{rT} \quad \leftarrow \text{present value of UA income}$$

$$F = (S + U) \cdot e^{rT} \quad \leftarrow \text{storage cost}$$

$$F = S e^{(r-q)T} \quad \leftarrow \text{dividend } (q)$$

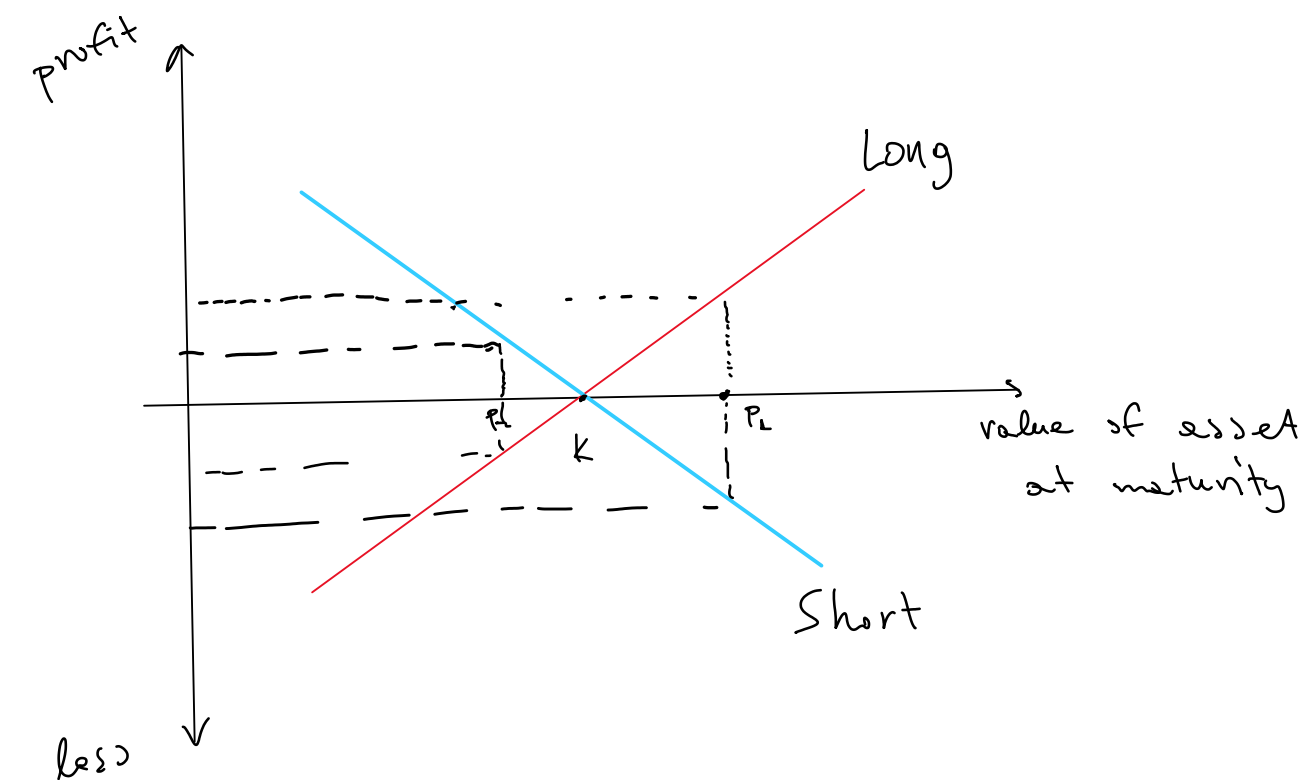
Example

USA : 7% interest rates for investment
 AUS : 5%

Spot price \$/AUD : 0.62 $\leftarrow 0.62 \$ = 1 \text{ AUD}$
 rate today

in two years future:

$$F = 0.62 \cdot e^{(0.07 - 0.05) \cdot 2} = 0.6453$$



Forward Price: $F = S \cdot e^{rT}$

Forward Value: $f = S - K e^{-rT}$

known income: $F = (S - I) \cdot e^{rT}$

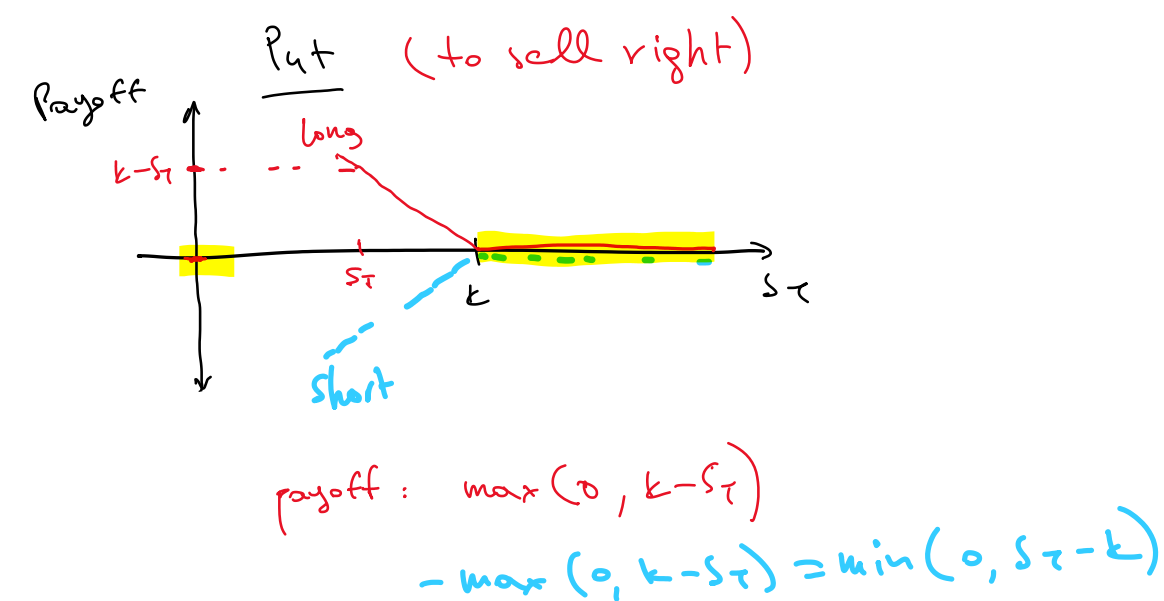
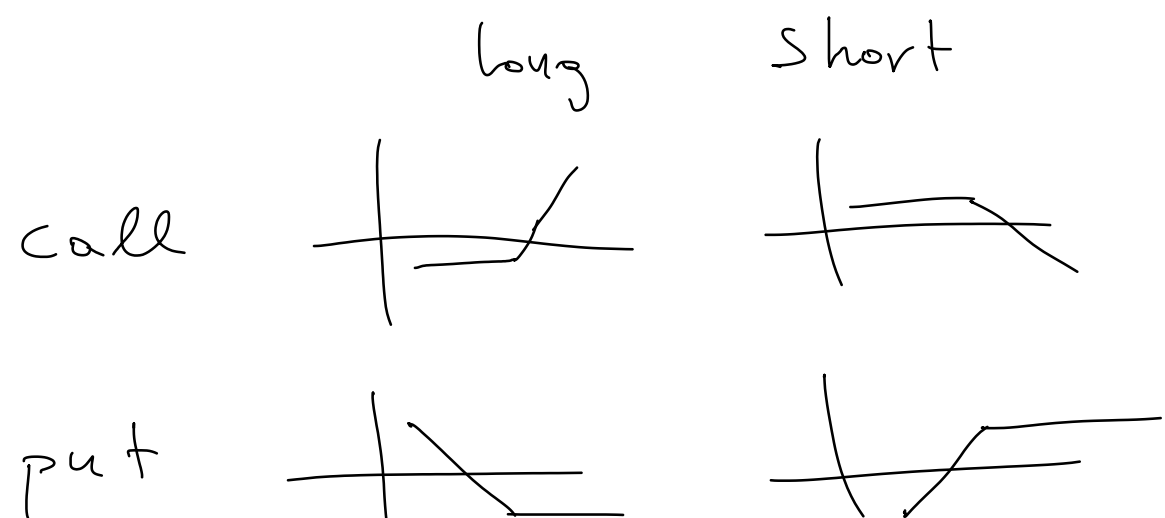
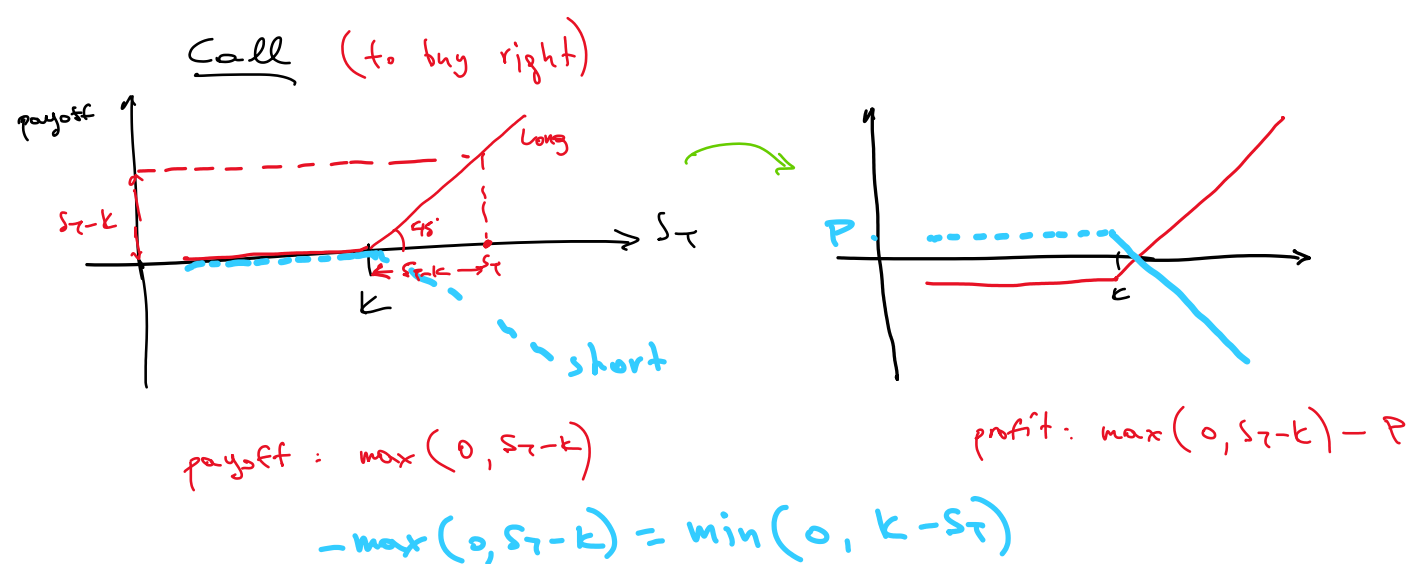
$$f = S - I - K e^{-rT}$$

known rates: $F = S \cdot e^{(r-q)T}$

$$f = S e^{-qT} - K e^{-rT}$$

Options Payoff Diagrams τ, K, S_τ

Call/Put option $\leftarrow \begin{matrix} \text{long} \\ \text{short} \end{matrix}$



Wednesday1

Wednesday, 17 March 2021

13:14

$$\boxed{NPV = PV - B_t}$$

To calculate NPV for given R

$$NPV = \frac{C_1}{(1+R)^1} + \frac{C_2}{(1+R)^2} + \frac{C_3}{(1+R)^3} + \dots + (-B_t)$$

$\xrightarrow{\text{spot price}} B_0$
 $\xrightarrow{\text{red}} (-B_t)$
 $\xrightarrow{\text{black in Excel}} \text{black in Excel}$

To calculate IRR (NPV = 0)

Manual

$$0 = \frac{C_1}{(1 + \text{semi annual IRR})^1} + \frac{C_2}{(1 + \text{IRR})^2} + \dots + (-B_t)$$

→ context

Excel

$$0 = \frac{C_1}{(1+x)^1} + \dots$$

$$\checkmark \left| \frac{6m}{(1+x)} \frac{6m}{(1+x)} - 1 = \text{Annual Rate} \right.$$

annual

$$1 - \frac{1}{1+x} = \text{Annual Rate}$$

$$(1.05)(1.05) = 1.1025$$

$$-1.00$$

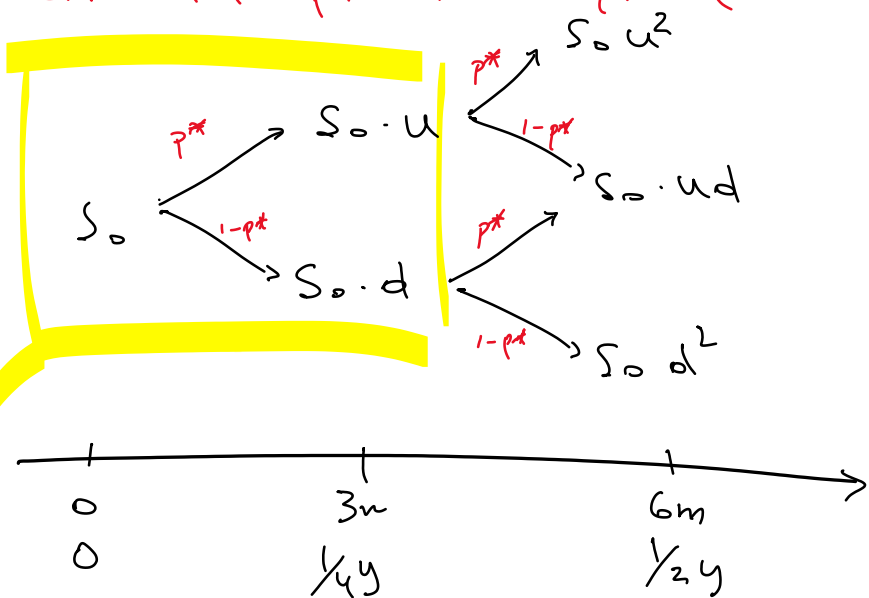
$$\hline 0.1025$$

S_0 : initial price of underlying asset (stock)
 S_T : price of asset at maturity
 T : maturity date (characteristic of option)
 K : strike price (characteristic of option)

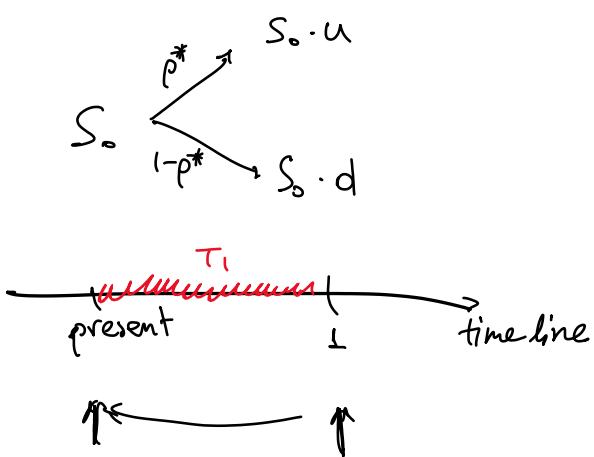
C : call
 P : put

C : European
 P : American

$r = 12\%$
 $S_0 = 20 \text{ £}$
European call ($K = 21 \text{ £}$, $T = 0.5 \text{ y}$)
 $U = 1 + 0.1 = 1.1$
 $d = 1 - 0.1 = 0.9$

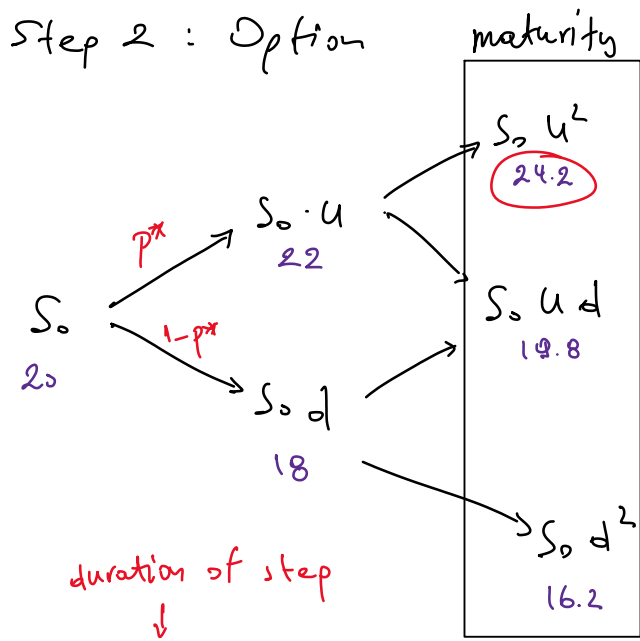
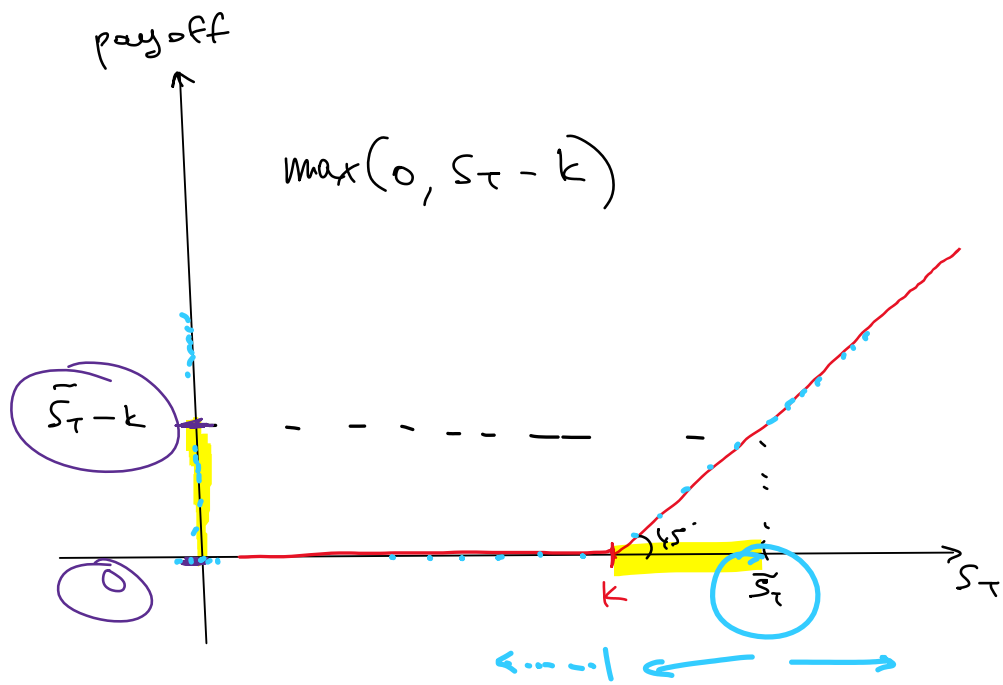


Step 1: Underlying Asset (price goes up/down) to set the binomial context



$E_1(S_1) = p^* \cdot S_0 \cdot U + (1-p^*) \cdot S_0 \cdot d$
 $E_0(S_1) = [p^* S_0 U + (1-p^*) S_0 d] \cdot e^{-rT_1} = S_0$

$[p^* \cdot u + (1-p^*) \cdot d] \cdot S_0 \cdot e^{-rT_1} = S_0$
 $p^* \cdot u + d - p^* \cdot d = e^{rT_1}$
 $p^* (u - d) = e^{rT_1} - d$
 $p^* = \frac{e^{rT_1} - d}{u - d}$ ← fair probability

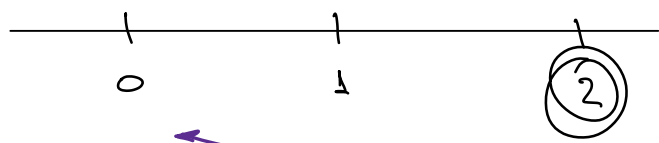


$\max(0, S_0 U^2 - K) = \max(0, 24.2 - 21) = \max(0, 3.2) = 3.2$
 $\max(0, \frac{19.8 - 21}{(-)}) = 0$

$e^{-rT_1} (p^* \cdot 3.2 + (1-p^*) \cdot 0) = 2.0257$
 $1.2823 = e^{-rT_2} (p^* \cdot 2.0257 + (1-p^*) \cdot 0)$
 $e^{-rT_2} (p^* \cdot 0 + (1-p^*) \cdot 0) = 0$

$K = 21 \text{ £}$

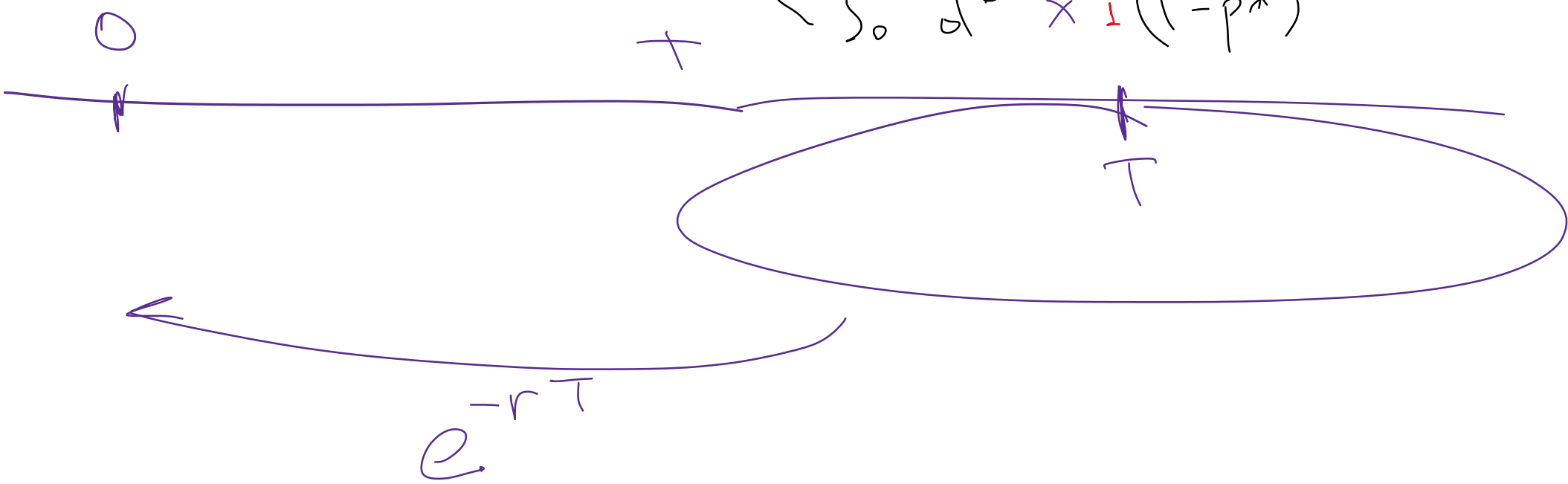
$p^* = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \cdot \frac{1}{4}} - 0.9}{1.1 - 0.9} = 0.6523$



$E_2(V_2) = V_2^u \cdot (p^*)^2 + V_2^d \cdot 2 \cdot p^* (1-p^*) + V_2^{dd} \cdot (1-p^*)^2$

where in time the expectation is calculated
 $E_0(V_2) = e^{-r(T_1+T_2)} \cdot (\dots)$

- $S_0 U^6 \times 1 \cdot (p^*)^6$
- $S_0 U^5 d^1 \times \dots$
- $S_0 U^4 d^2 \times \dots$
- $S_0 U^3 d^3 \times \dots$
- $S_0 U^2 d^4 \times \dots$
- $S_0 U^1 d^5 \times \dots$
- $S_0 d^6 \times 1 \cdot (1-p^*)^6$



Thursday1

Thursday, 18 March 2021

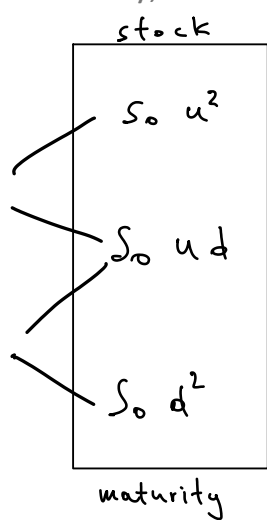
13:01

reminder
for
binomial

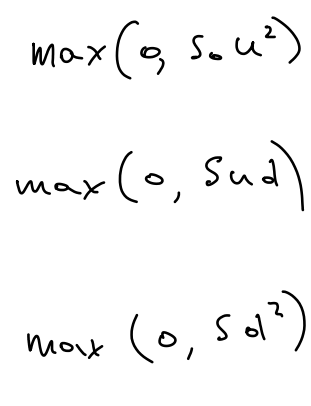
$$\text{Prob} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

n: number of trials

k: number of successes



value of option (call)



number of increases $\leftarrow$ number of decreases $\rightarrow$

$$\frac{(\#u + \#d)!}{(\#u)! (\#d)!}$$

number of steps

$$= \binom{n}{\#u} (p^*)^{\#u} (1-p^*)^{\#d}$$

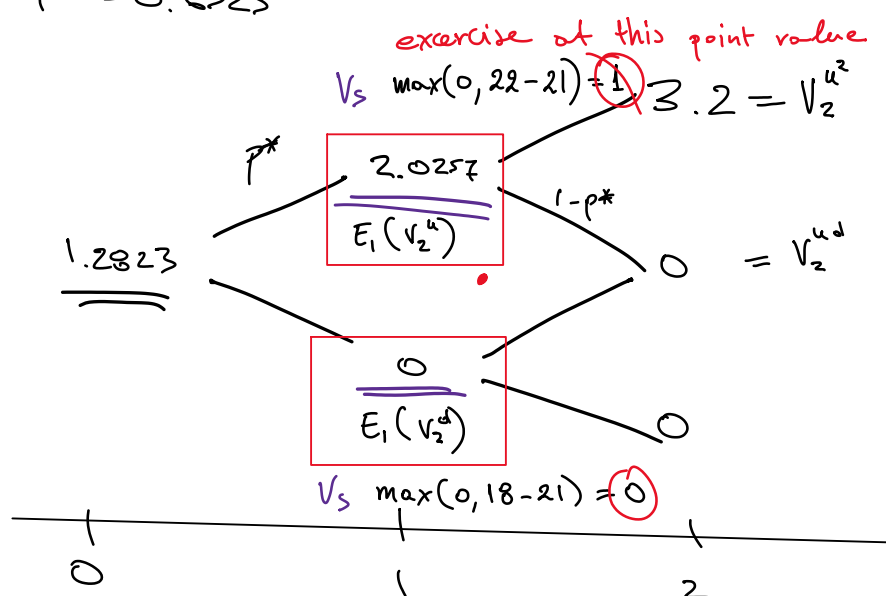
number of increases

$$E_2(V_2) = \sum_{j,i} \text{prob.} \cdot V_2^{j,i}$$

$$E_0(V_2) = e^{-r(T_1+T_2)} \cdot E_2(V_2)$$

Option tree (see wednesday3)

$$p^* = 0.523$$



American allows the option to be exercised in intermediate steps

At each point two things are compared

- Value of option at certain point
- Expected value at certain point

$$S_0 = 30$$

European Put $\Rightarrow \max(0, K - S_T)$

$$K = 35$$

$$T = 10m \left\{ \begin{array}{l} T_1 + T_2 = 10 \\ T_1 = T_2 \end{array} \right\} \Rightarrow T_1 = T_2 = 5$$

$$u \sim 20\% \left\{ \begin{array}{l} 1 + 0.2 = 1.2 \\ 1 - 0.2 = 0.8 \end{array} \right.$$

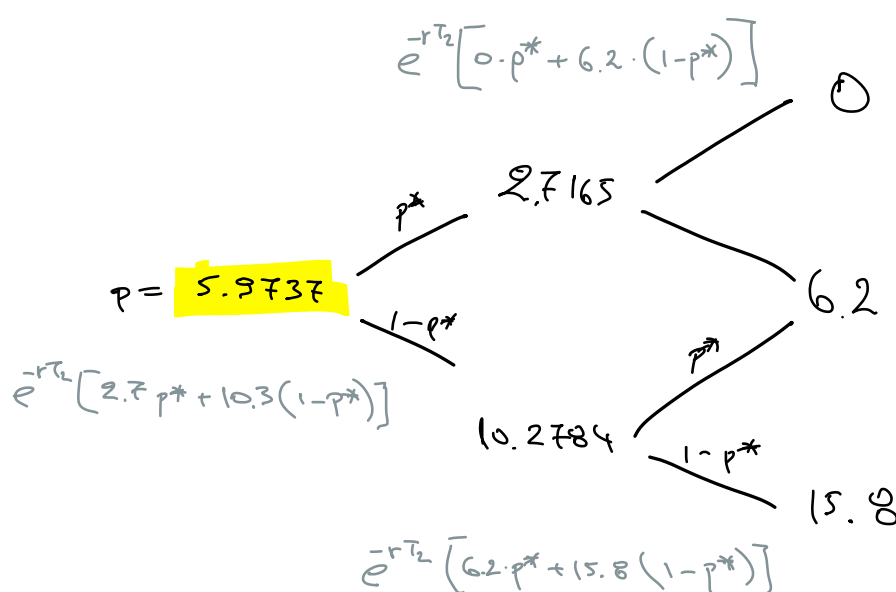
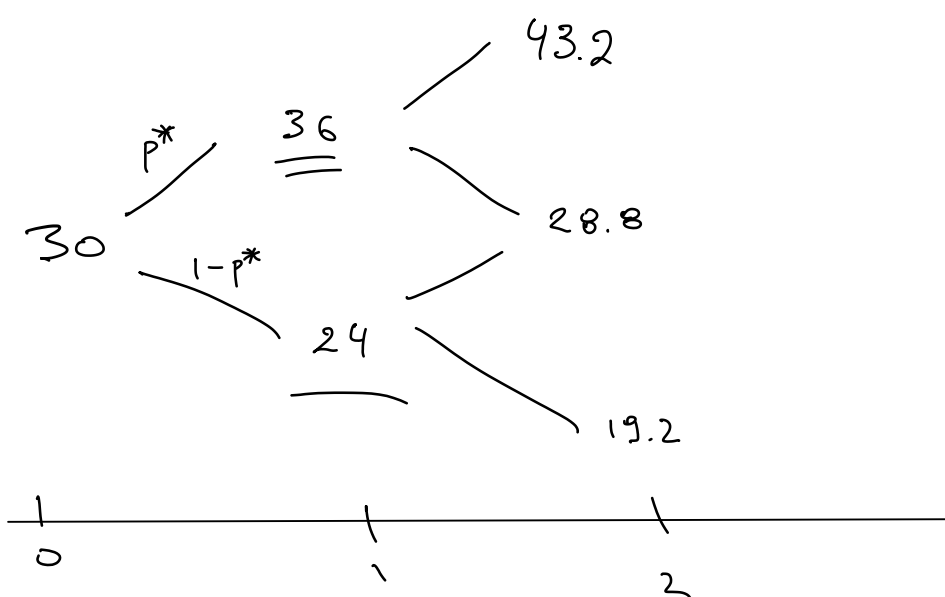
$$r = 5\%$$

one interval in years

$$p^* = \frac{e^{rT_1} - d}{u - d} = \frac{e^{0.05 \cdot \frac{5}{12}} - 0.8}{1.2 - 0.8}$$

$$p^* = 0.5526$$

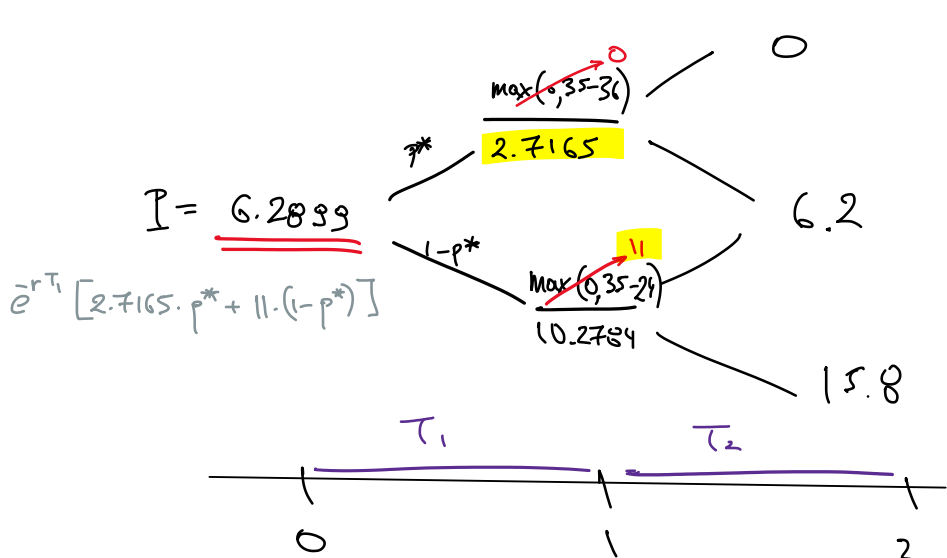
$$(1-p^*) = 0.4474$$



Exel
needed for American

$$5.9737 = e^{-rT} \left[0 \cdot (p^*)^2 + 6.2 \cdot \frac{2!}{1!1!} (p^*)^1 (1-p^*)^1 + 15.8 \cdot (p^*)^0 (1-p^*)^2 \right]$$

American Option Case



Asset Price: $S =$

Value of option: $V =$

Expectation: $E =$

European

Calculated

only at maturity

All way back

American

Calculated

All way back

All way back

Compare
choose

Ito lemma

Friday, 19 March 2021

20:25

Examples:

$$X_t = 4 B_t^2 \xrightarrow{B_t=y} f(t, y) = 4y^2 \quad \frac{\partial f}{\partial y} = 8y \xrightarrow{y=B_t} f_2 = 8 \cdot B_t$$

Apply Ito's Lemma:

$$\frac{\partial^2 f}{\partial y^2} = 8 \xrightarrow{} f_{22} = 8$$

$$dX_t = (f_1 + \frac{1}{2} f_{22}) dt + (f_2) dB_t$$

$$\left. \begin{array}{l} f_1 = 0 \\ f_2 = 8 B_t \\ f_{22} = 8 \end{array} \right\} \Rightarrow \underbrace{dX_t = 4 dt + 8 B_t dB_t}_{\text{SDE}}$$

$$X_t = 1 + 2t + 3t^2 + e^{B_t^4} \xrightarrow{} f(t, y) = 1 + 2t + 3t^2 + \underline{e^{y^4}}$$

$$f_1 = 2 + 6t$$

$$f_2 = 4 B_t^3 \cdot e^{B_t^4}$$

$$f_{22} = 12 B_t^2 \cdot e^{B_t^4} + 4 B_t^3 \cdot 4 B_t^3 \cdot e^{B_t^4} = (12 B_t^2 + 16 B_t^6) e^{B_t^4}$$

$$\frac{\partial(e^{y^4})}{\partial y} \cdot e^{y^4}$$

$\Rightarrow$ Ito's lemma ...

$$dX_t = (2 + 6t + (12 B_t^2 + 16 B_t^6) \cdot e^{B_t^4}) dt + (4 B_t^3 \cdot e^{B_t^4}) dB_t$$

$$\rightarrow X_t = X_0 \cdot e^{\underbrace{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}} \xrightarrow{} f(t, y) = X_0 \cdot e^{\underbrace{(\mu - \frac{1}{2}\sigma^2)t + \sigma y}} = \underbrace{X_0 e^{At}}_{X_0 \cdot e^{\sigma y} A e^{At}} \cdot \underbrace{e^{\sigma y}}_{X_0 e^{At} \sigma \cdot e^{\sigma y}}$$

$$f_1 = X_0 (\mu - \frac{1}{2}\sigma^2) e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}$$

$$f_2 = X_0 \sigma e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}$$

$$f_{22} = X_0 \sigma^2 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}$$

$$dX_t = (f_1 + \frac{1}{2} f_{22}) dt + f_2 dB_t$$

$$= \left[\underline{X_0 (\mu - \frac{1}{2}\sigma^2)} + \frac{1}{2} \underline{X_0 \sigma^2} \right] \underline{e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}} \cdot dt + \underline{X_0 \sigma} \underline{e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}} dB_t$$

$$\Rightarrow dX_t = \underline{X_0 \cdot e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}} \left[\left(\cancel{\mu - \frac{1}{2}\sigma^2} + \cancel{\frac{1}{2}\sigma^2} \right) dt + \sigma dB_t \right]$$

$$\Rightarrow dX_t = X_t (\mu dt + \sigma dB_t) \Rightarrow \frac{dX_t}{X_t} = \mu dt + \sigma dB_t$$

BSM Model

Friday, 19 March 2021

20:23

Example (notes / excel)

$$S_0 = 30$$

$$K = 35$$

$$T = 10m = 0.83$$

$$r = 5\%$$

$$n = 2$$

$$u = 1.2$$

$$d = 0.8$$

$$\Delta t = \frac{0.83}{2} = \frac{T}{n}$$

1. Find d_1, d_2

2. Go to table (or Excel) for finding $N(+d_1), N(+d_2)$

3. Do calculations (C, P)

variance of underlying asset assumption $\sigma = 0.31$

$$\left(\begin{array}{l} u = e^{\sigma \sqrt{\Delta t}} \\ d = e^{-\sigma \sqrt{\Delta t}} \end{array} \right) \left\{ \begin{array}{l} \ln u = \sigma \sqrt{\Delta t} \\ \ln d = -\sigma \sqrt{\Delta t} \end{array} \right\} \left\{ \begin{array}{l} \sigma = \frac{\ln u}{\sqrt{\Delta t}} = \frac{\ln(1.2)}{\sqrt{0.415}} = \frac{0.1823}{0.644} = 0.283 \\ \sigma = \frac{-\ln d}{\sqrt{\Delta t}} = \frac{-\ln(0.8)}{\sqrt{0.415}} = \frac{0.223}{0.644} = 0.3462 \end{array} \right.$$

$$d_1 = \frac{\ln(30/35) + (0.05 + \frac{0.31^2}{2})0.83}{0.31 \sqrt{0.83}} = -0.2738$$

$$d_2 = \frac{\ln(30/35) + (0.05 - \frac{0.31^2}{2})0.83}{0.31 \sqrt{0.83}} = -0.5477$$

$$N(-d_2) = N(0.5477) = N(0.54) + 0.77 \left(N(0.55) - N(0.54) \right) =$$

$$N(-d_1) = N(0.2738) = N(0.27) + 0.38 \left(N(0.28) - N(0.27) \right) =$$

$$P = 35 e^{-0.05 \cdot 0.83} \cdot 0.7080 - 0.6079$$

$$P = \$5.5329$$