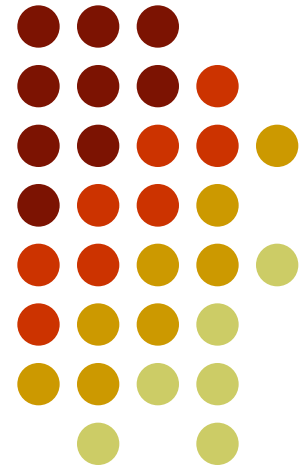


Forward and Futures Prices



Consumption vs Investment Assets



- Investment assets are assets held by significant numbers of people purely for investment purposes
(Examples: gold, silver)
- Consumption assets are assets held primarily for consumption
(Examples: copper, oil)

Short Selling



- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way



Short Selling (continued)

- At some stage you must buy the securities so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives
- There may be a small fee for borrowing the securities



Example

- You short 100 shares when the price is \$100 and close out the short position three months later when the price is \$90
- During the three months a dividend of \$3 per share is paid
- What is your profit?
- What would be your loss if you had bought 100 shares?

Notation for Valuing Futures and Forward Contracts



S_0 : Spot price today

F_0 : Futures or forward price today

T : Time until delivery date

r : Risk-free interest rate for
maturity T



An Arbitrage Opportunity?

- Suppose that:
 - The spot price of a non-dividend-paying stock is \$40
 - The 3-month forward price is \$43
 - The 3-month US\$ interest rate is 5% per annum
- Is there an arbitrage opportunity?



Another Arbitrage Opportunity?

- Suppose that:
 - The spot price of nondividend-paying stock is \$40
 - The 3-month forward price is US\$39
 - The 1-year US\$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

The Forward Price



If the spot price of an investment asset is S_0 and the futures price for a contract deliverable in T years is F_0 , then

$$F_0 = S_0(1+r)^T$$

where r is the T -year risk-free rate of interest.

In our examples, $S_0 = 40$, $T = 0.25$, and $r = 0.05$ so that

$$F_0 = 40(1.05)^{0.25} = 40.5$$

When Interest Rates are Measured with Continuous Compounding



$$F_0 = S_0 e^{rT}$$

This equation relates the forward price and the spot price for any investment asset that provides no income and has no storage costs

If Short Sales Are Not Possible..



Formula still works for an investment asset because investors who hold the asset will sell it and buy forward contracts when the forward price is too low

When an Investment Asset Provides a Known (fixed) Income



$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the
income during life of forward contract



When an Investment Asset Provides a Known Yield

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)



Valuing a Forward Contract

- A forward contract is worth zero (except for bid-offer spread effects) when it is first negotiated
- Later it may have a positive or negative value
- Suppose that K is the delivery price and F_0 is the forward price for a contract that would be negotiated today



Valuing a Forward Contract

- By considering the difference between a contract with delivery price K and a contract with delivery price F_0 we can deduce that:

- The value, f , of a long forward contract is

$$(F_0 - K)e^{-rT}$$

- the value of a short forward contract is

$$(K - F_0)e^{-rT}$$

Forward vs Futures Prices



- When the maturity and asset price are the same, forward and futures prices are usually assumed to be equal. (Eurodollar futures are an exception)
- When interest rates are uncertain they are, in theory, slightly different:
 - A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
 - A strong negative correlation implies the reverse



Stock Index

- Can be viewed as an investment asset paying a dividend yield
- The futures price and spot price relationship is therefore

$$F_0 = S_0 e^{(r-q)T}$$

where q is the dividend yield on the portfolio represented by the index during life of contract



Stock Index (continued)

- For the formula to be true it is important that the index represent an investment asset
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio
- The Nikkei index viewed as a dollar number does not represent an investment asset

Index Arbitrage



- When $F_0 > S_0 e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures
- When $F_0 < S_0 e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index

Index Arbitrage

(continued)



- Index arbitrage involves simultaneous trades in futures and many different stocks
- Very often a computer is used to generate the trades
- Occasionally simultaneous trades are not possible and the theoretical no-arbitrage relationship between F_0 and S_0 does not hold

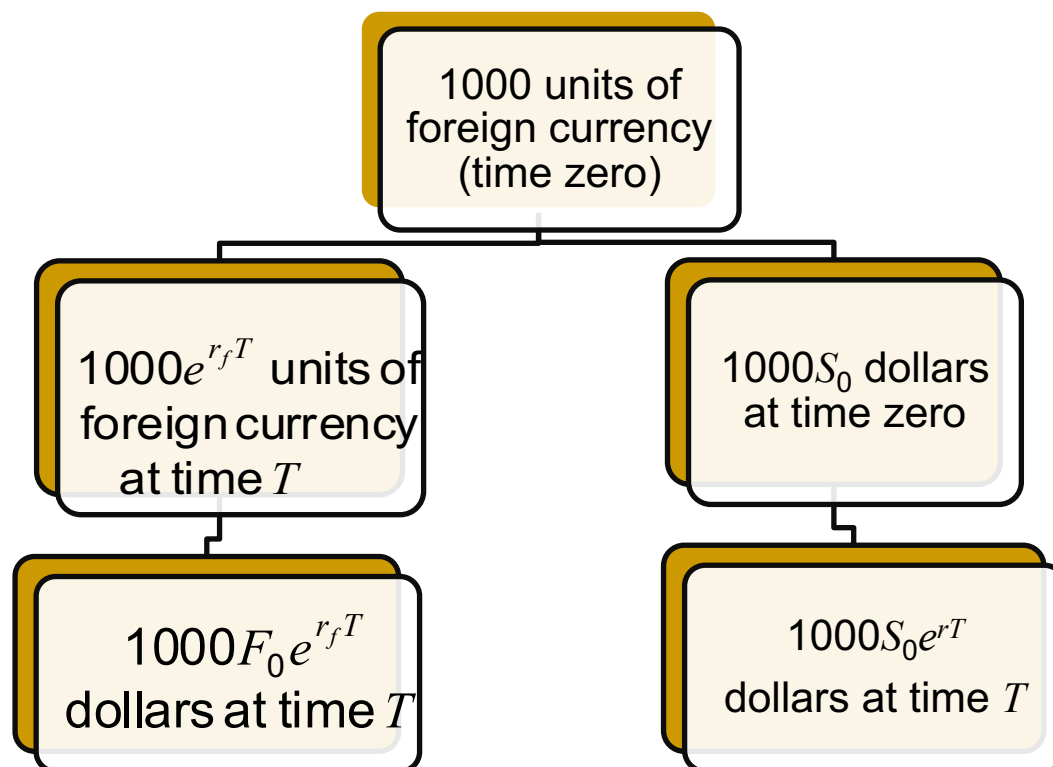
Futures and Forwards on Currencies



- A foreign currency is analogous to a security providing a yield
- The yield is the foreign risk-free interest rate
- It follows that if r_f is the foreign risk-free interest rate

$$F_0 = S_0 e^{(r-r_f)T}$$

Explanation of the Relationship Between Spot and Forward



Consumption Assets: Storage is Negative Income



$$F_0 \leq S_0 e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.
Alternatively,

$$F_0 \leq (S_0 + U) e^{rT}$$

where U is the present value of the storage costs.

The Cost of Carry



- The cost of carry, c , is the storage cost plus the interest costs less the income earned
- For an investment asset $F_0 = S_0 e^{cT}$
- For a consumption asset $F_0 \leq S_0 e^{cT}$
- The convenience yield on the consumption asset, y , is defined so that
$$F_0 = S_0 e^{(c-y)T}$$

Futures Prices & Expected Future Spot Prices



- Suppose k is the expected return required by investors in an asset
- We can invest $F_0 e^{-rT}$ at the risk-free rate and enter into a long futures contract to create a cash inflow of S_T at maturity
- This shows that

$$F_0 e^{-rT} e^{kT} = E(S_T)$$

or

$$F_0 = E(S_T) e^{(r-k)T}$$



Futures Prices & Future Spot Prices (continued)

No Systematic Risk	$k = r$	$F_0 = E(S_T)$
Positive Systematic Risk	$k > r$	$F_0 < E(S_T)$
Negative Systematic Risk	$k < r$	$F_0 > E(S_T)$

Positive systematic risk: stock indices

Negative systematic risk: gold (at least for some periods)