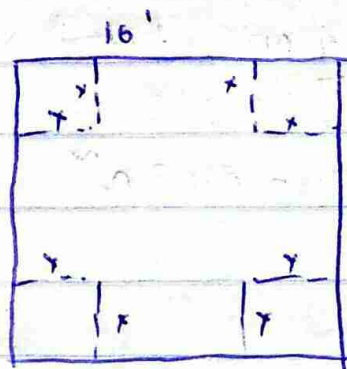


Optimization Assignment

7 / July / 2020



Goal is to maximize the volume

$$\text{Vol, } v = l \times w \times h, \quad l = 21 - 2x, \quad w = 16 - 2x, \quad \text{height, } h = x$$

$$v(x) = (21 - 2x) \times (16 - 2x) \times x = 4x^3 - 74x^2 + 336x$$

$$v'(x) = 12x^2 - 148x + 336 = 4(x-3)(8x-28)$$

When $v'(x) = 0$, we have critical points

$$x = 3, \quad x = \frac{28}{3} \quad x = \frac{28}{3} \text{ will result negative width}$$

The second derivative test proves at $x = 3$, volume is maximum

$$\text{width} = 16 - 2x = 16 - 6 = 10''$$

$$\text{length} = 21 - 2x = 21 - 6 = 15''$$

$$\text{height} = x = 3''$$

2. Let n be number of passengers over 400, total income

$$R(n) = (400+n)[40 - 0.05n]$$

$$R(n) = 16000 - 20n + 40n - 0.05n^2$$

$$\frac{dR}{dn} = 20 - 0.1n$$

$$\frac{dR}{dn} = 0 \quad : \quad 20 - 0.1n = 0$$

$$20 = 0.1n$$

$$n = 200$$

$$\text{Passengers will be } 400 + n = 400 + 200$$

$$= \boxed{600 \text{ passengers}}$$

2. Let L be length, W be width

$$2L + 2W = 20$$

$$L + W = 10$$

$$W = 10 - L$$

$$\text{Area} = L \times W = L \times 10 - L$$

$$A = 10L - L^2$$

$$\frac{dA}{dL} = 10 - 2L$$

set derivative to zero

$$10 = 2L$$

$$L = 5 \text{ ft}$$

$$W = 10 - 5 = 5$$

$$\begin{aligned} \text{Area} &= 5 \times 5 \\ &= 25 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned}
 4 \quad \text{Volume, } V &= L \times W \times H \\
 &= x \times x \times y \\
 &= x^2 w
 \end{aligned}$$

$$\text{Girth} + \text{Length} = 4x + w = 108 \quad 120$$

$$\cancel{w = 108 - 4x} \quad w = 120 - 4x$$

$$V = V(x) = x^2 (120 - 4x)$$

$$V(x) = x^2 (120 - 4x) = 120x^2 - 4x^3$$

$$\frac{dV}{dx} = 240x - 12x^2$$

$$\frac{dV}{dx} = 240x - 12x^2 = 0$$

$$\begin{aligned}
 240x &= 12x^2 \\
 x &= \frac{240}{12} = 20''
 \end{aligned}$$

Base dimensions: 20" by 20"

$$\text{Height} = 120 - 4x$$

$$= 120 - 4(20)$$

$$= 120 - 80$$

$$= 40''$$

5. Let x and y be dimensions of enclosure.
 x with y being side made of blocks.
 $A = xy = 600$

$2x + y$ costs \$ per ft

y costs \$14

Cost function

$$C = 7(2x + y) + 14y$$

$$= 14x + 14y + 14y$$

$$= 14x + 28y$$

$$A = xy = 600, \quad x = \frac{600}{y}$$

$$C = 14\left(\frac{600}{y}\right) + 28y = \frac{8400}{y} + 28y$$

$$C' = \frac{-8400}{y^2} + 28 = 0$$

$$y = 20 \text{ ft}, \quad x = 30 \text{ ft}$$

We minimize cost at 20 feet by 30 feet