Instructions: *You need to show enough details of your work to receive full credit. When asked to draw a graph, label things clearly. (I recommend you to draw graphs by hand. Do not copy and paste any graphs from the lecture notes, otherwise you will receive zero credit.) Total points are 100.*

1 Short Questions [10]

1. Based on the 2004 paper by Edward Prescott, what is the main reason for the differences in average hours worked between European countries and the U.S.? Explain.

2. What is the Laffer Curve? What is the shape of the Laffer Curve? Draw it in a graph (You do not need to draw it accurately, just the shape in general.) What are the tax policy implications of the Laffer Curve?

3. What is the permanent income hypothesis? What does it say about the responses of consumption to income changes? Is this prediction of the permanent income hypothesis consistent with the data in reality?

2 The Two-Period Model of Households [65]

Consider a household who lives for two periods: the current period and the future period. The income of the household is y in the current period and y' in the future period. In both periods, the household has to pay lump-sum taxes to the government. The tax liability is t for the current period and t' for the future period. A credit market is available to the household, so the household can lend (save) or borrow in the current period with an interest rate r. The household will receive or pay back the principal plus interest in the future period. The amount of savings in the current period is denoted by s. (A negative s implies the household is borrowing.) The household's preferences over the current consumption c and the future consumption c' are represented by a utility function $U(c, c')$, which is both monotonic and concave.

1. In the (c, c') space, draw two of the household's indifference curves with different utility levels. What are the properties of the indifference curves? Label clearly which one of the two indifference curves represents a higher utility level.

2. What is the household's budget constraint for the current period?

3. What is the household's budget constraint for the future period?

4. Derive the household's lifetime budget constraint by combining the budget constraints of both periods and eliminating the savings s. (Hint: For each period budget constraint, try to put savings s on one side and everything else on the other side, and then connect the two resulting inequalities through s. In the end, it is better to put all the terms about expenditures c and c' on one side and all the terms about income $y - t$ and $y' - t'$ on the other side.)

5. Are there other constraints that the household's choice must obey? If so, what are them?

6. In the (c, c') space, draw the set of feasible choices of the household. Label clearly the following: (1) the slope of the budget line; (2) the intercepts on the vertical and horizontal axes; (3) the area of feasible choices; (4) the endowment point E.

7. What is the household's optimization problem? Be clear about the following: (1) the objective function; (2) the choice variables; (3) the constraints if any.

8. Assume there is an interior solution to the household's problem. In the (c, c') space, use indifference curves and the set of feasible choices to find the solution to the household's optimization problem, i.e., the optimal choice of the household.

9. Based on your graph in the last question, write down the optimality conditions that must be satisfied at the household's optimal choice. (Hint: you should have two conditions.)

10. Suppose both the current and future consumption goods are normal goods, discuss and explain **without any graphs** what will happen to the household's current consumption c, future consumption c' and savings s if there is

    (a) an increase in current income y;

    (b) an increase in future income y';

    (c) an increase in interest rate r, assuming the household is a lender;

    (d) an increase in interest rate r, assuming the household is a borrower.

11. Suppose both the current and future consumption goods are normal goods, and **the substitution effect is stronger than the income effect**. In the (c, c') space, show

graphically the effects of the following changes to the household's optimal choice. If applicable, be clear about which part of the change is due to the income effect and which part is due to the substitution effect.

(a) an increase in future income y';

(b) an increase in interest rate r, assuming the household is a lender;

(c) an increase in interest rate r, assuming the household is a borrower.

For Question 12 to 15, let the utility function of the household be

$$U(c, c') = \ln(c) + \beta \ln(c'),$$

where β is a parameter between 0 and 1, and assume that there is always an interior solution to the household's problem.

12. What is the marginal rate of substitution of current consumption for future consumption $MRS_{c,c'}$ given this utility function? How does it change with c and c'?

13. Solve the household's optimization problem with the lifetime budget constraint. That is, find the household's optimal current consumption c and future consumption c' in terms of (y, y', t, t', r, β).

14. From your answer to the last question, what is the household's savings s in the current period? Your answer should be in terms of (y, y', t, t', r, β).

15. The parameter β in the utility function governs how patient the household is. From your answer to the last two questions, how do the household's current consumption c, future consumption c', and savings s respond to an increase of β (i.e., when the household becomes more patient)? (Hint: The value of $\frac{x}{1+x}$ increases with x when x is between 0 and 1.)

3 The Two-Period Model of Firms [25]

Consider a firm that produces the consumption good in two periods: the current period and the future period. The production function in the current period is

$$Y = zF(K, N)$$

where Y is the current output, z is the current total factor productivity, K is the current capital stock, and N is the current labor hired. The production function in the future period is

$$Y' = z'F(K', N')$$

where Y' is the future output, z' is the future total factor productivity, K' is the future capital stock, and N' is the future labor hired.

The firm is a price-taker, and the real wage of labor is w in the current period and w' in the future period. The interest rate in the credit market is r.

The capital stock in the current period K is given (i.e., not chosen by the firm), but the firm can invest I units of output in the current period to increase its capital stock in the future period K'. Capital depreciates after being used in production, and the depreciation rate is d. So the future capital stock is determined by the following law of motion for capital.

$$K' = (1-d)K + I.$$

At the end of the future period, all the capital left after the production $(1-d)K'$ can be converted one-for-one back into the consumption good, which can be sold in the same way as the output produced.

1. What is the firm's profit in the current period π in units of the current consumption good? (Hint: Profit=Revenue−Cost.)

2. What is the firm's profit in the future period π' in units of the future consumption good? (Hint: Profit=Revenue−Cost.)

3. Write down the firm's optimization problem. Be clear about the following: (1) the objective function (You should replace the output Y and Y' with the corresponding production functions here.); (2) the choice variables; (3) the constraints if any.

For Question 4 to 5, let the production functions in the current and future periods be

$$Y = zK^{\alpha},$$
$$Y' = z'K'^{\alpha},$$

where $\alpha \in (0,1)$ is a parameter. These production functions imply that no labor is used in the production, and hence there is no labor cost for the firm in either period. Other settings are the same as before.

4. Solve the firm's problem with these production functions. That is, find the firm's optimal investment I and the future capital stock K' in terms of parameters.

5. Based on your answer to the last question, how does the firm's investment decision respond to

   (a) an increase in the interest rate r;

(b) an increase in the current capital stock K;

(c) an increase in the future total factor productivity z'.