

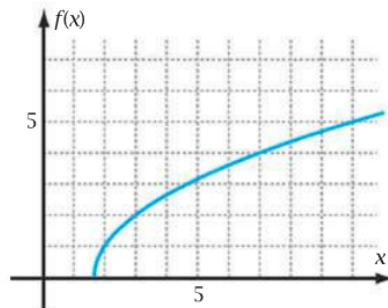
Name: _____ Date: _____

Math 110A – Calculus I – Fall 2020 - Instructor: Jean Nanjo
PRACTICE FINAL EXAM

You have 2 hours to complete this test. Graphing calculators are allowed but must be approved by the instructor (no computer algebra systems, cell phones, etc.). You are allowed to use a half-sheet of handwritten notes. Show all work!

1. The graph of $f(x)$ is shown at right.

- a) Estimate the value of $f'(7)$.
b) Draw the graphical representation of your answer from part (a) onto the graph at right.



2. The table below shows another function, $f(x)$. Estimate $f'(36)$. Show your calculations.

x	$f(x)$
30	74
32	77
34	83
36	88
38	90
40	91
42	89

3. Sketch a graph of a function for which the following is true:

a) $\lim_{x \rightarrow 3^-} f(x) = 4$

b) $\lim_{x \rightarrow 3^+} f(x) = \infty$

4. Use the definition of derivative to find $f'(5)$, where $f(x) = x^3$.

5. Find the equation of a line tangent to the graph of $y = x \cos x$ at $x = \pi/3$.

6. Find the values of a and b that make f differentiable at $x = 2$.

$$\begin{cases} ax^2 + 1, & x < 2 \\ -x^2 + 6x + b, & x \geq 2 \end{cases}$$

7. Calculate $f'(x)$.

a) $f(x) = \sqrt{3x - 5}$

b) $f(x) = \sin^{-1} x^3$

8. Calculate $\frac{dy}{dx}$.

a) $y = e^{2x} \cos 3x$

b) $y = \frac{\ln x}{\tan x}$

9. Calculate y' .

$$\cos(xy) + y^2 - \ln x = 1$$

10. Differentiate the function.

a) $y = \log_3(x^2 + 1)$

b) $y = 5^x$

11. a) State Rolle's theorem.

b) Sketch a graph that illustrates the conclusion of the mean value theorem.

For questions 12 through 14, let $f(x) = x^2\sqrt{4 - x^2}$.

12. For what values of x is the graph of f increasing? Decreasing? At a local max? Local min? Use calculus to justify your answer.

13. For what values of x is the graph of f concave up? Concave down? At a point of inflection? Use calculus to justify your answer.

14. Use the second derivative test to find all local max/min values for f .

15. State whether each function has horizontal and/or vertical asymptote(s), if any. Use calculus to find the equation(s) for the asymptote(s).

a) $f(x) = \frac{x^2 - 2x}{x + 1}$

b) $g(x) = \frac{3x^2 - x + 5}{x^2 - 4}$

16. Find the particular solution to the differential equation $f'(x) = (t + 1)^{-2} + e^{-2t}$, $f(0) = 2$

17. The table below shows selected values for $f'(x)$. Use a Left Riemann Sum with three subintervals to estimate the net change in f from $x = 31$ to $x = 42$.

x	$f'(x)$
31	74
32	77
34	83
36	88
38	90
40	91
42	89

18. Evaluate using the Fundamental Theorem of Calculus.

(a) $\int_1^3 \left(x^2 + \frac{1}{x}\right) dx$

(b) $\int_0^{\pi/4} \sec^2 x \, dx$

19. Find dy/dx :

(a) $y = \int_0^x \sqrt{1 + t^3} \, dt$

(b) $y = \int_{2x}^{x^2} \sin t \, dt$

20. Evaluate: $\int \frac{\sin(x-3)}{\cos^3(x-3)} dx$

21. Evaluate using Fundamental Theorem of Calculus: $\int_e^{2e} \frac{dx}{x(\ln x)^2}$