

Econ 100B: Economic Analysis – Macroeconomics
Problem Set #5 – Solutions
Due Date: October 9, 2020

General Instructions:

- Please upload a PDF of your problem set to Gradescope by 11:00 pm.
- **Late homework will not be accepted.**
- Please put your name and student ID at the upper right corner of the front page.

1. Beginning with the second equation for Y in the middle of slide 22 of lecture 9, carry out the algebra needed to obtain the equation for the IS curve shown on Slide 25.

Solution: Beginning with the IS curve

$$Y = \bar{C} + mpc \times Y^d + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L, \quad (1)$$

we see that the substitution $Y^d = Y - T = Y - \bar{T}$ yields

$$Y = \bar{C} + mpc \times (Y - \bar{T}) + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L. \quad (2)$$

Collecting terms in Y we find that

$$Y(1 - mpc) = \bar{C} - mpc \times \bar{T} + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L, \quad (3)$$

which, after dividing both sides by $(1 - mpc)$ and moving $-mpc \times \bar{T}$ to the end of the sum of autonomous components, yields the IS curve:

$$Y = \frac{(\bar{C} + \bar{I} + \bar{G} + \bar{NX} - mpc \times \bar{T})}{(1 - mpc)} - \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)} r_L \quad (4)$$

Replacing the lending rate r_L with the lending rate function

$$r_L = r + s_{\text{term}} + s_{\text{credit}} \quad (5)$$

we get

$$\frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)} r_L = \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)} (r + s_{\text{term}} + s_{\text{credit}}) \quad (6)$$

$$= \frac{\bar{S}}{(1 - mpc)} + \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)} r \quad (7)$$

where

$$\bar{S} = (\zeta_C + \zeta_I + \zeta_{NX}) (s_{\text{term}} + s_{\text{credit}}) \quad (8)$$

and

$$Y = \frac{(\bar{C} + \bar{I} + \bar{G} + \bar{NX} - mpc \times \bar{T} - \bar{S})}{(1 - mpc)} - \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{(1 - mpc)} r \quad (9)$$

2. If the IS curve does not shift when autonomous investment increases by \$150 MM, autonomous taxes increase by \$425 MM, and no other autonomous components change,
- (a) what is the value of the expenditure multiplier?

Solution: The expenditure multiplier is given by

$$\frac{1}{1 - mpc} \quad (10)$$

where mpc is the marginal propensity to consume. Thus, if we know the mpc we know the expenditure multiplier. Given that the IS curve did not shift following the changes in these autonomous components it follows that these changes have offset (or cancelled) one another. Referring to Equation (??) we see that this implies that

$$\Delta \bar{I} - mpc \times \Delta \bar{T} = 0 , \quad (11)$$

or

$$\$150 \text{ MM} - mpc \times \$425 \text{ MM} = 0 , \quad (12)$$

or

$$mpc = \frac{\$150 \text{ MM}}{\$425 \text{ MM}} = 0.3529 . \quad (13)$$

and it follows that the expenditure multiplier is

$$\frac{1}{1 - mpc} = \frac{1}{1 - 0.3529} = 1.545 \quad (14)$$

- (b) what is the value of the tax multiplier?

Solution: The tax multiplier is also related to the marginal propensity to consume by

$$- \frac{mpc}{1 - mpc} = - \frac{0.3529}{1 - 0.3529} = -0.545 \quad (15)$$

3. In our derivation of the IS curve we made the simple assumption that taxes are completely autonomous: $T = \bar{T}$. A more realistic tax function would be one where taxes increase with income as described by $T = \bar{T} + tY$. Follow the steps you undertook in question (1) with this more realistic tax function to derive a new IS-curve equation.

Solution: Returning to the IS curve

$$Y = \bar{C} + mpc \times Y^d + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L , \quad (16)$$

but now substituting with $Y^d = Y - T = Y - \bar{T} - tY$ yields

$$Y = \bar{C} + mpc \times (Y - \bar{T} - tY) + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L . \quad (17)$$

Collecting terms in Y we find that

$$Y [1 - mpc(1 - t)] = \bar{C} - mpc \times \bar{T} + \bar{I} + \bar{G} + \bar{NX} - (\zeta_C + \zeta_I + \zeta_{NX}) r_L , \quad (18)$$

which, after dividing both sides by $[1 - mpc(1 - t)]$ and moving $-mpc \times \bar{T}$ to the end of the sum of autonomous components, yields the IS curve:

$$Y = \frac{(\bar{C} + \bar{I} + \bar{G} + \bar{NX} - mpc \times \bar{T})}{[1 - mpc(1 - t)]} - \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{[1 - mpc(1 - t)]} r_L \quad (19)$$

As in question 1 above, this is sufficient to answer the question. To include $\bar{S}$ we would proceed as we did in that solution and obtain

$$Y = \frac{(\bar{C} + \bar{I} + \bar{G} + \bar{NX} - mpc \times \bar{T} - \bar{S})}{[1 - mpc(1 - t)]} - \frac{(\zeta_C + \zeta_I + \zeta_{NX})}{[1 - mpc(1 - t)]} r \quad (20)$$

4. Beginning with the IS curves shown at the top of slide 33 of lecture 9, write out the steps needed to derive the IS curve $\tilde{y} = -\zeta_y \tilde{r}$.

Solution: Given

$$Y = \bar{Y} - \zeta_Y r \quad (21)$$

and

$$Y^P = \bar{Y} - \zeta_Y r^* , \quad (22)$$

subtracting the second from the first and dividing by Y^P yields

$$\frac{Y - Y^P}{Y^P} = -\frac{\zeta_Y}{Y^P} (r - r^*) . \quad (23)$$

The output gap can be written

$$\frac{Y - Y^P}{Y^P} = \frac{Y}{Y^P} - 1 \approx \ln(Y) - \ln(Y^P) \equiv y - y^P \quad (24)$$

where the relationship $\ln(x) = x - 1 + \dots$ has been used. Thus the gap-form of the IS curve becomes

$$y - y^P = -\frac{\zeta_Y}{Y^P} (r - r^*) \quad (25)$$

which we can write as

$$y - y^P = -\zeta_y (r - r^*) \quad (26)$$

since ζ_y is the sensitivity of y with respect to r . Finally, using the “tilde” notation to represent gaps, the gap form of the IS curve becomes

$$\tilde{y} = -\zeta_y \tilde{r} \quad (27)$$

where

$$\tilde{y} = y - y^P \quad (28)$$

and

$$\tilde{r} = r - r^* . \quad (29)$$