

Econ 100B: Economic Analysis – Macroeconomics
Problem Set #6 – Solutions
Due Date: October 16, 2020

General Instructions:

- Please upload a PDF of your problem set to Gradescope by 11:00 pm.
- **Late homework will not be accepted.**
- Please put your name and student ID at the upper right corner of the front page.

1. Comparing the output and unemployment gaps shown on slide 25 of lecture 10:

- (a) Would you characterize the direction of these fluctuations as generally procyclical, countercyclical, or acyclical? Why?

Solution: The direction of these fluctuations is generally countercyclical because almost all of the movements in the output gap have a corresponding opposite direction of movement by the unemployment gap.

- (b) Many of the relationships in our macroeconomic models are linear; two variables are related by the equation for a straight line $y = mx + b$. What is the relationship between (i) the sign of the slope m and (ii) the cyclicity (pro or counter) seen in a time series? Briefly explain.

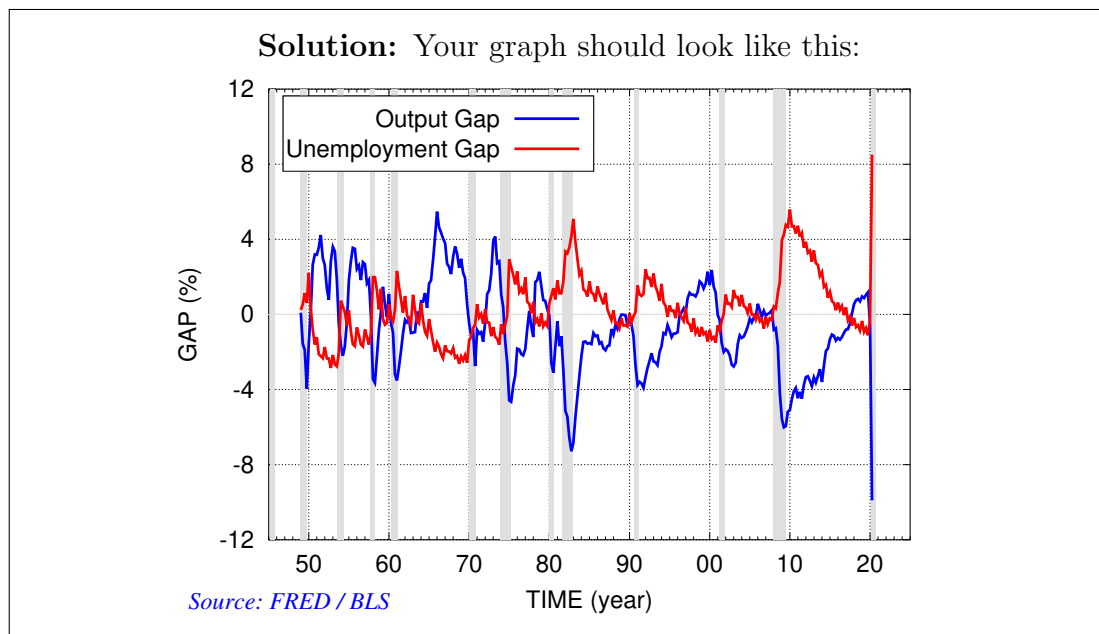
Solution: If variables y and x are *procyclical* then the direction of change of one is the same as the other: if one increases so does the other and if one decreases so does the other. This would imply that $m > 0$. Similarly, if variables y and x are *countercyclical* then the direction of change in one is the opposite of that of the other: if one variable increases the other decreases and if one variable decreases the other increases. This would imply that $m < 0$.

2. The goal of this exercise is to estimate Okun's law using data from the Federal Reserve. Download the following time-series from FRED:¹

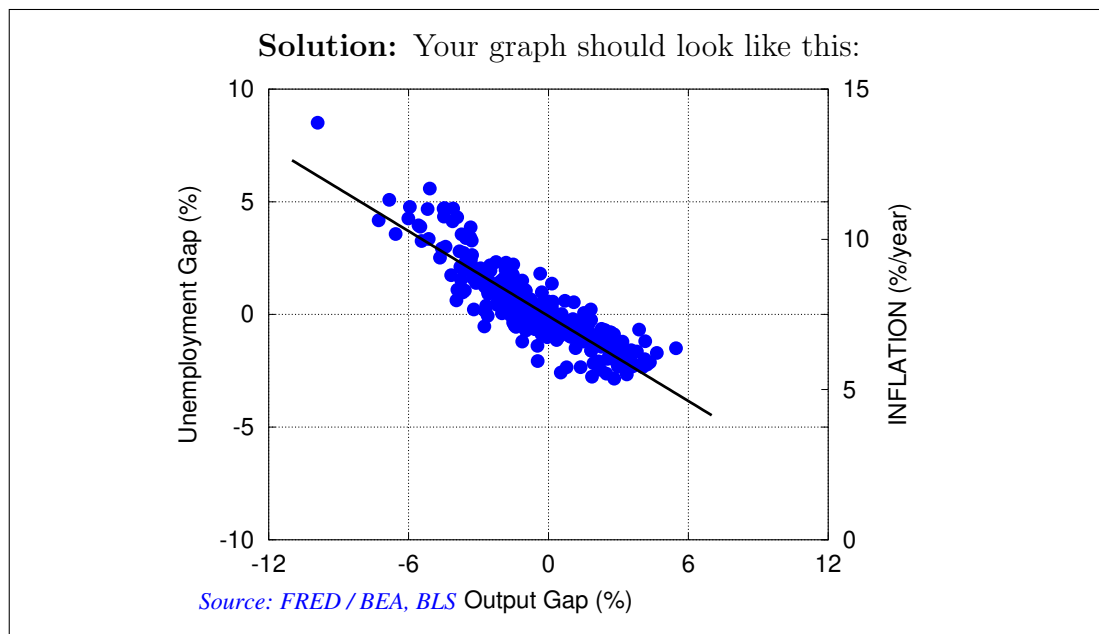
- The civilian unemployment rate (UNRATENSA): U .
- The natural rate of unemployment (NROU): U_N .
- Real gross domestic product (GDPC1): Y .
- Real potential gross domestic product (GDPPOT): Y^P .

¹For purposes of this exercise we will ignore the seasonality inconsistency in the GDP measures.

- (a) From these time series calculate and plot the unemployment gap, $U - U_N$, and the output gap $100 \times (Y - Y^P)/Y^P$ as a function of time (and on a quarterly basis) from the first quarter of 1949 to the most recent data available. Your graph should look similar to the graph on slide 25 of lecture 10.



- (b) Create a scatter plot of the unemployment gap as a function of the output gap. Your graph should look *similar to* the graph on slide 26 of lecture 10. *Note: you'll need to extend the axes shown in class to get the most recent data point onto the graph.*



- (c) Perform a linear regression on the data in the graph you generated for Question (2b) and show the regression result as a line on your graph. (*Note: The regression method you used in the analysis of the Cobb-Douglas data can be used here.*)

Solution: The regression result shown as a black line in the scatter plot above (obtained using the trendline option in Excel) is $\tilde{u} = -0.6286\tilde{y} - 0.0717$ ($R^2 = 0.77$) where $\tilde{u}$ denotes the unemployment gap and $\tilde{y}$ denotes the output gap.

- (d) Does your regression analysis support the equation for Okun's Law discussed in class? Why?

Solution: Yes. The equation for Okun's law discussed in class is $\tilde{u} \approx -0.6\tilde{y}$ (cf. slide 26 of lecture 10). The functional form of the equation – a linear relationship between the two gaps with a (possibly) zero y-intercept and slope of about -0.6 – is supported by this regression.

3. Write out the steps in the derivation of the optimal rate rule given on slides 6–9 of lecture 11. In your derivation include comments that explain what you are doing.

Solution: We begin with the Phillips curve

$$\pi_t = \pi_{t-1} + \gamma (y_t - y^P) \quad (1)$$

the IS curve

$$y_t - y^P = -\zeta_y (r_{t-1} - r^*) \quad (2)$$

and the central-bank loss function

$$L = (y_t - y^P)^2 + \beta (\pi_t - \pi^T)^2 . \quad (3)$$

The minimization of the loss function with respect to y_t and π_t can be reduced to a minimization with respect to y_t alone by substituting the Phillips curve into the loss function as follows

$$L = (y_t - y^P)^2 + \beta (\pi_{t-1} + \gamma (y_t - y^P) - \pi^T)^2 . \quad (4)$$

Taking the derivative with respect to y_t

$$\frac{dL}{dy_t} = 2 (y_t - y^P) + 2\beta\gamma (\pi_{t-1} + \gamma (y_t - y^P) - \pi^T) \gamma , \quad (5)$$

then reversing the Philips curve substitution

$$\frac{dL}{dy_t} = 2 (y_t - y^P) + 2\beta\gamma (\pi_t - \pi^T) , \quad (6)$$

and setting the derivative to zero to find the minimum yields

$$(y_t - y^P) = -\beta\gamma (\pi_t - \pi^T) . \quad (7)$$

If we expand the inflation term using the Phillips curve we get

$$(y_t - y^P) = -\beta\gamma (\pi_{t-1} + \gamma (y_t - y^P) - \pi^T) \quad (8)$$

which, because the dynamic IS curve expresses y_t in terms of r_{t-1} , will set us up to have a rate function in terms of inflation at the same time. Dividing through by $-\gamma\beta$ and collecting terms in the output gap on the left hand side we find that

$$\frac{-1}{\beta\gamma} (y_t - y^P) = (\pi_{t-1} + \gamma (y_t - y^P) - \pi^T) \quad (9)$$

and

$$-\left(\gamma + \frac{1}{\beta\gamma}\right) (y_t - y^P) = (\pi_{t-1} - \pi^T) . \quad (10)$$

Substituting the gap form of the IS equation gives us an equation relating the rate gap to the output gap we get

$$\zeta_y \left(\gamma + \frac{1}{\beta\gamma} \right) (r_{t-1} - r^*) = (\pi_{t-1} - \pi^T) \quad (11)$$

from which our optimal rate equation – or *monetary policy rule* – emerges as

$$r_{t-1} = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\beta\gamma} \right)} (\pi_{t-1} - \pi^T) \quad (12)$$

which holds for all times $t - 1$ and can be written in $r(t)$ form as

$$r(t) = r^* + \frac{1}{\zeta_y \left(\gamma + \frac{1}{\beta\gamma} \right)} (\pi(t) - \pi^T) . \quad (13)$$

4. The current Fed Funds rate band is 0.0%/year – 0.25%/year. If the Federal reserve wanted to set their policy rate to 0.10%/year using a 2-day reverse repo transaction, how much would a commercial bank pay to buy back \$65 million worth of bonds?

Solution: From our expression for the repo rate

$$r_{\text{repo}} = \left(\frac{P_{t_1} - P_{t_0}}{P_{t_0}} \right) \left(\frac{365}{t_1 - t_0} \right) \quad (14)$$

$$= \left(\frac{P_{t_1}}{P_{t_0}} - 1 \right) \left(\frac{365}{t_1 - t_0} \right) \quad (15)$$

so

$$\left(\frac{t_1 - t_0}{365} \right) r_{\text{repo}} = \frac{P_{t_1}}{P_{t_0}} - 1 \quad (16)$$

and

$$\frac{P_{t_1}}{P_{t_0}} = 1 + \left(\frac{t_1 - t_0}{365} \right) r_{\text{repo}} \quad (17)$$

which gives

$$P_{t_1} = P_{t_0} \left(1 + \left(\frac{t_1 - t_0}{365} \right) r_{\text{repo}} \right) . \quad (18)$$

Substituting the given levels of the variables we get

$$P_{t_1} = \$30 \text{ million} \times \left(1 + \left(\frac{2}{365} \right) \times 0.0010 \right) \quad (19)$$

or

$$P_{t_1} = \$65,000,356.16 \quad (20)$$