

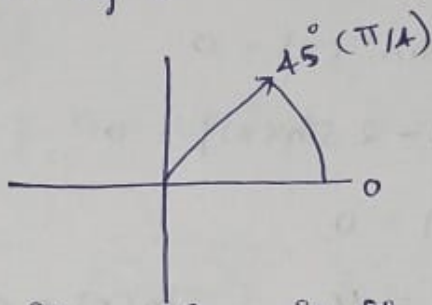
- 1) Find the exact value of $\cos\left(\frac{\theta}{2}\right)$ for the given conditions
 $\csc(\theta) = \frac{4}{3}$; $0^\circ < \theta < 90^\circ$

Ans Quadrant for θ ?

Given $0^\circ < \theta < 90^\circ$

Divide above interval by 2

$$0 < \frac{\theta}{2} < 45^\circ$$



Thus cosine is positive, since it lies in the first quadrant

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \text{--- ①}$$

Given,

$$\operatorname{cosec} \theta = \frac{4}{3}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \Rightarrow \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{3}{4}\right)^2 + \cos^2 x = 1$$

$$\cos^2 x = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos x = \frac{\sqrt{7}}{4} \quad \text{--- ②}$$

Sub ② in ①

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \frac{\sqrt{7}}{4}}{2}} = \frac{\sqrt{2} \sqrt{4 + \sqrt{7}}}{4}$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{2} \sqrt{4 + \sqrt{7}}}{4}$$

2) Find the exact solutions to the equation that are in the interval $[0, 2\pi)$

$$\sin(x) + \cos(2x) = 1$$

Ans

$$\cos(2x) = 1 - 2\sin^2(x)$$

$$\Rightarrow \sin(x) + 1 - 2\sin^2(x) = 1$$

$$\sin(x) - 2\sin^2(x) = 0$$

$$\sin(x) [1 - 2\sin(x)] = 0$$

$$1 - 2\sin(x) = 0$$

$$\sin(x) = 1/2, \quad \sin(x) = 0$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ and } \pi$$

3) Find the exact value of $\tan(2\theta)$ for the given value of θ

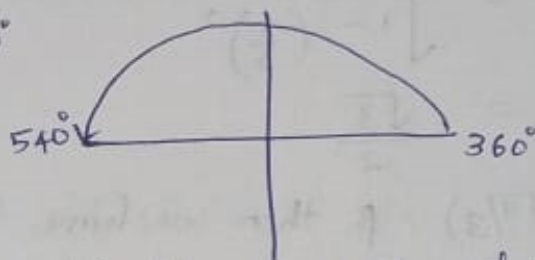
$$\sec(\theta) = -6; \quad 180^\circ < \theta < 270^\circ$$

Ans

$$\text{Given } 180^\circ < \theta < 270^\circ$$

Multiply the above interval by 2

$$360^\circ < 2\theta < 540^\circ$$



Since the tangent lies between 360° and 540° i.e., 1st and second quadrant, the tangent can take both positive (1st quad) and negative value [2nd quad].

$$\tan(2\theta) = \pm \frac{2 \cos x \sqrt{1 - \cos^2 x}}{2 \cos^2 x - 1} \quad \text{--- (1)}$$

Given,

$$\sec \theta = -6$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{-6} = -\frac{1}{6} \quad \text{--- (2)}$$

Sub (2) in (1)

$$\tan(2\theta) = \pm \frac{2 \left(-\frac{1}{6}\right) \sqrt{1 - \left(-\frac{1}{6}\right)^2}}{2 \left(-\frac{1}{6}\right)^2 - 1}$$

$$\tan(2\theta) = \pm \frac{\sqrt{35}}{24}$$

4)

Find the exact value of the expression

$$\sin(\cos^{-1}(-1/2) - \tan^{-1}(\sqrt{7}/3))$$

Ans

Let $\cos^{-1}(-1/2) = \alpha$, then $\cos \alpha = -1/2$

$$\begin{aligned} \text{and } \sin \alpha &= \sqrt{1 - \cos^2 \alpha} \\ &= \sqrt{1 - \left(-\frac{1}{2}\right)^2} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Let $\tan^{-1}(\sqrt{7}/3) = \beta$ then we have $\tan \beta = \sqrt{7}/3$

$$\text{and } \cos \beta = \frac{1}{\sec \beta} = \frac{1}{\sqrt{1 + \tan^2 \beta}} = \frac{1}{\sqrt{1 + 7/9}} = \frac{3}{4}$$

$$\text{and } \sin \beta = \tan \beta \times \cos \beta = \frac{\sqrt{7}}{3} \times \frac{3}{4} = \frac{\sqrt{7}}{4}$$

Hence $\sin[\cos^{-1}(-1/2) - \tan^{-1}(\sqrt{7}/3)]$

$$= \sin(\alpha - \beta)$$

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \frac{\sqrt{3}}{2} \times \frac{3}{4} - \left[-\frac{1}{2}\right] \times \frac{\sqrt{7}}{4}$$

$$= \frac{3\sqrt{3} + \sqrt{7}}{8}$$