

Question #2

Let the dimensions of the box be x ft $\cdot$ x ft $\cdot$ y ft

$$\text{Volume} = 36 \text{ ft}^3 \Rightarrow x^2 y = 36 \Rightarrow y = \frac{36}{x^2}$$

$$\begin{aligned}\text{cost, } C(x, y) &= 8x^2 + 5(4xy) \\ &= 8x^2 + 20xy\end{aligned}$$

$$\Rightarrow C(x, y) = 8x^2 + 20x \left(\frac{36}{x^2} \right) \Rightarrow 8x^2 + 20 \left(\frac{36}{x} \right) \Rightarrow 8x^2 + \frac{720}{x}$$

$$C'(x, y) = 16x - \frac{720}{x^2}$$

$$C''(x, y) = 16 + \frac{1440}{x^3}$$

For minimum cost $C'(x) = 0$, $C''(x) > 0$

$$16x - \frac{720}{x^2} = 0 \Rightarrow 16x = \frac{720}{x^2} \Rightarrow x^3 = \frac{720}{16} \Rightarrow x = 6.71$$

$$y = \frac{36}{6.71^2} = 1.47$$

$$C''(6.71) = 16 + \frac{1440}{6.71^3} > 0 \quad \cancel{C'(6.71) = 0}$$

$$C(6.71) = 8(6.71)^2 + \frac{720}{6.71} = 360 + 107.33 = 467.33$$

Therefore, dimensions of the box for minimum cost are
6.71 ft $\cdot$ 6.71 ft $\cdot$ 1.47 ft. Minimum cost is \$467.33.