

Question #3

a) $n_1=10$ $n_2=10$ $n_3=10$ $n_4=10$ $m=4$
 $\bar{x}_1=2$ $\bar{x}_2=2$ $\bar{x}_3=6$ $\bar{x}_4=8$ $n = \sum_{i=1}^m n_i = 40$
 $s_1^2=20$ $s_2^2=20$ $s_3^2=20$ $s_4^2=20$

$$\text{Grand mean } (\bar{\bar{x}}) = \frac{\sum_{i=1}^m n_i \bar{x}_i}{\sum_{i=1}^m n_i} = \frac{10(2) + 10(2) + 10(6) + 10(8)}{10 + 10 + 10 + 10}$$

$$= \frac{2 + 2 + 6 + 8}{4} = \frac{18}{4} = 4.5 = \bar{\bar{x}}$$

$$SS_{Tr} = \sum_{i=1}^m n_i (\bar{x}_i - \bar{\bar{x}})^2$$

$$= 10 [(2-4.5)^2 + (2-4.5)^2 + (6-4.5)^2 + (8-4.5)^2]$$

$$= 10 (2.5^2 + 2.5^2 + 1.5^2 + 3.5^2)$$

$$SS_{Tr} = 270$$

$$SS_{Er} = \sum_{i=1}^m (n_i - 1) s_i^2 = 9 (20^2 + 20^2 + 20^2 + 20^2)$$

$$= 9 \times 4 \times 20^2 = 14,400.$$

$$SS_{Er} = 14,400.$$

$$MS_{Tr} = \frac{SS_{Tr}}{m-1} = \frac{270}{4-1} = \frac{270}{3} = 90$$

$$MS_{Er} = \frac{SS_{Er}}{n-m} = \frac{14,400}{40-4} = \frac{14,400}{36} = 400$$

$$SS_{T_0} = 270 + 14,400 = 14,670$$

$$Df_{Tr} = m-1 = 4-1 = 3$$

$$Df_{Er} = n-m = 40-4 = 36$$

$$Df_{T_0} = n-1 = 40-1 = 39$$

Variation	DF	SS	MS
Treatment	3	270	90
Error	36	14,400	400
Total	39	14,670	

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$$b) \text{ F-test, } F = \frac{MS_{Tr}}{MS_{Er}} = \frac{90}{400} = 0.225$$

$$\left. \begin{array}{l} \text{F-test} = 0.225 \\ \text{Df}_{Tr} = 3 \\ \text{Df}_{Er} = 36 \end{array} \right\} \text{ p-value} = 0.8784$$

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

H_a : At least two means are not equal

$$\alpha = 0.05$$

Since $\text{p-value} = 0.8784 > 0.05$, it falls in the non-rejection region. Therefore, fail to reject the null hypothesis and conclude that there is not significant evidence of the four factor levels have the same means.