

Question #15Product Rule

$$\begin{aligned}
 a) \frac{d}{dx}(\sin x \sqrt{x}) &= \frac{d}{dx}(\sin x) \sqrt{x} + \frac{d}{dx}(\sqrt{x}) \sin x \\
 &= \cos x \sqrt{x} + \frac{1}{2\sqrt{x}} \sin x \\
 &= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x
 \end{aligned}$$

Region of differentiability: $\sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$

$$\begin{aligned}
 b) \frac{d}{dx} \left(\frac{\cos x}{1 + \cos x} \right) &= \frac{\frac{d}{dx}(\cos x)(1 + \cos x) - \frac{d}{dx}(1 + \cos x) \cos x}{(1 + \cos x)^2} \quad \text{Quotient Rule} \\
 &= \frac{(-\sin x)(1 + \cos x) - (-\sin x) \cos x}{(1 + \cos x)^2} \\
 &= \frac{\sin(2x) - 2 \sin x (1 + \cos x)}{2(1 + \cos x)^2}
 \end{aligned}$$

Region of differentiability: $\frac{\sin(2x) - 2 \sin x (1 + \cos x)}{2(1 + \cos x)^2}$

$$\begin{aligned}
 c) \int_1^s \frac{1}{s} ds &= \ln|s|_1^s \\
 &= \ln|s|
 \end{aligned}$$

Region of differentiability: $\ln|s|$

$$\begin{aligned}
 d) \int_1^{\cos x} |t| dt &= \left[\frac{t|t|}{2} \right]_1^{\cos x} \\
 &= \frac{1}{2} \cos x |\cos x| - \frac{1}{2}
 \end{aligned}$$

Region of differentiability: $\frac{1}{2} \cos x |\cos x| - \frac{1}{2}$