Quiz 7A: Chapter 7

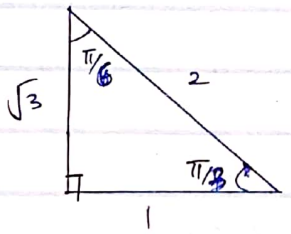
1 $\sin \frac{5\pi}{4} \cos \frac{13\pi}{12} - \sin \frac{13\pi}{12} \cos \frac{5\pi}{4}$

$\Rightarrow \sin(A - B) = \sin A \cos B - \cos A \sin B$

$\therefore \quad A = \frac{5\pi}{4}, \quad B = \frac{13\pi}{12}$

$\Rightarrow \sin\left(5\pi/4 - 13\pi/12\right) = \sin\left(\pi/6\right)$

$\Rightarrow \sin 5\pi/4 \cos 13\pi/12 - \sin 13\pi/12 \cos 5\pi/4 = \sin\left(\pi/6\right)$



$\Rightarrow \sin\left(\pi/6\right) = 1/2$

$= \frac{1}{2}$

2. $\tan\left(\frac{7\pi}{6} + \frac{7\pi}{4}\right) = \frac{\tan\left(7\pi/6\right) + \tan\left(7\pi/4\right)}{1 - \tan\left(7\pi/6\right)\tan\left(7\pi/4\right)}$

$\tan\left(7\pi/6\right) = \tan\left(\pi/6\right) = \frac{\sqrt{3}}{3}$

$\tan\left(7\pi/4\right) = \tan\left(3\pi/4\right) = -1$

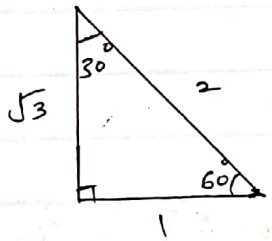
$\Rightarrow \tan\left(7\pi/6 + 7\pi/4\right) = \frac{\sqrt{3}/3 - 1}{1 + \left(\frac{\sqrt{3}}{3}\right)} = -2 + \sqrt{3}$

3. $2 \sin\left(\frac{\pi}{6}\right) \cos\left(\pi/6\right)$

$\Rightarrow \sin 2\theta = 2\sin\theta \cos\theta$

$\theta = \frac{\pi}{6}$; $2\theta = \frac{\pi}{3}$

$\Rightarrow 2\sin\left(\pi/6\right)\cos\left(\pi/6\right) = \sin\left(\pi/3\right)$



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

4. $2\sin^2 x + \sqrt{3}\sin x = 0$ for $0 \leq x \leq 2\pi$

Let $\sin x$ be x

$\Rightarrow 2x^2 + \sqrt{3}x = 0$

$x(2x + \sqrt{3}) = 0$

$x = 0$

$2x + \sqrt{3} = 0$; $2x = -\sqrt{3}$

$x = -\frac{\sqrt{3}}{2}$

$\sin x = 0$; $x = 0, \pi, 2\pi$

$\sin x = -\frac{\sqrt{3}}{2}$; $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

Solutions: $x = 0$, $x = \pi$, $x = 2\pi$, $x = \frac{4\pi}{3}$, $x = \frac{5\pi}{3}$

5. (a) $\sin \alpha = \cos(\alpha - \pi/2)$

(b) $\sin \alpha = -\sin(\alpha - \pi)$

(c) $\sin \alpha = -\cos(\alpha - 3\pi/2)$

6. Derivation of $\sin 2\theta$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

let $B = A$

$\sin(A+A) = \sin A \cos A + \cos A \sin A$

$\therefore \sin(2A) = 2 \sin A \cos A$

Similarly: $\sin 2\theta = 2 \sin\theta \cos\theta$

Derivation of $\cos 2\theta$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

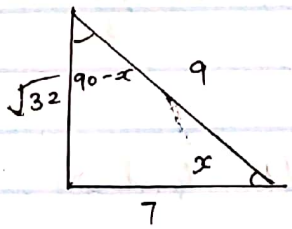
let $B = A \Rightarrow \cos(A+A) = \cos A \cos A \mp \sin A \sin A$

$\cos(2A) = \cos^2 A \mp \sin^2 A$

$= \cos^2 A - \sin^2 A$

$\therefore \cos 2\theta = \cos^2\theta - \sin^2\theta$

7. $\cos x = \frac{7}{9}$



$\sin 2x = 2 \sin x \cos x$

$\sin x = \frac{\sqrt{32}}{9}$

$\sin 2x = 2 \times \frac{\sqrt{32}}{9} \times \frac{7}{9} = \frac{56\sqrt{2}}{81}$

8. $\frac{\cos(x-y)}{\cos(x+y)} = \frac{1+\tan x \tan y}{1-\tan x \tan y}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\Rightarrow \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$

$= \frac{\frac{\cos x \cos y}{\cos x \cos y} + \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}{\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}$

$\Rightarrow \frac{\sin x}{\cos x} = \tan x \; ; \; \frac{\sin y}{\cos y} = \tan y$

$\therefore \Rightarrow \frac{1+\tan x \tan y}{1-\tan x \tan y}$

LHS = RHS

9. $\sin x = \frac{-\sqrt{3}}{2} \qquad 0 \le x \le 2\pi$

(a) 2 solutions ; $x = \frac{4\pi}{3}, \; x = \frac{5\pi}{3}$

(b) 3rd and 4th Quadrants

(c) $x = \frac{4\pi}{3}, \; x = \frac{5\pi}{3}$

(o) $f(x) = 8\sin x \cos x$

$\sin 2x = 2\sin x \cos x$

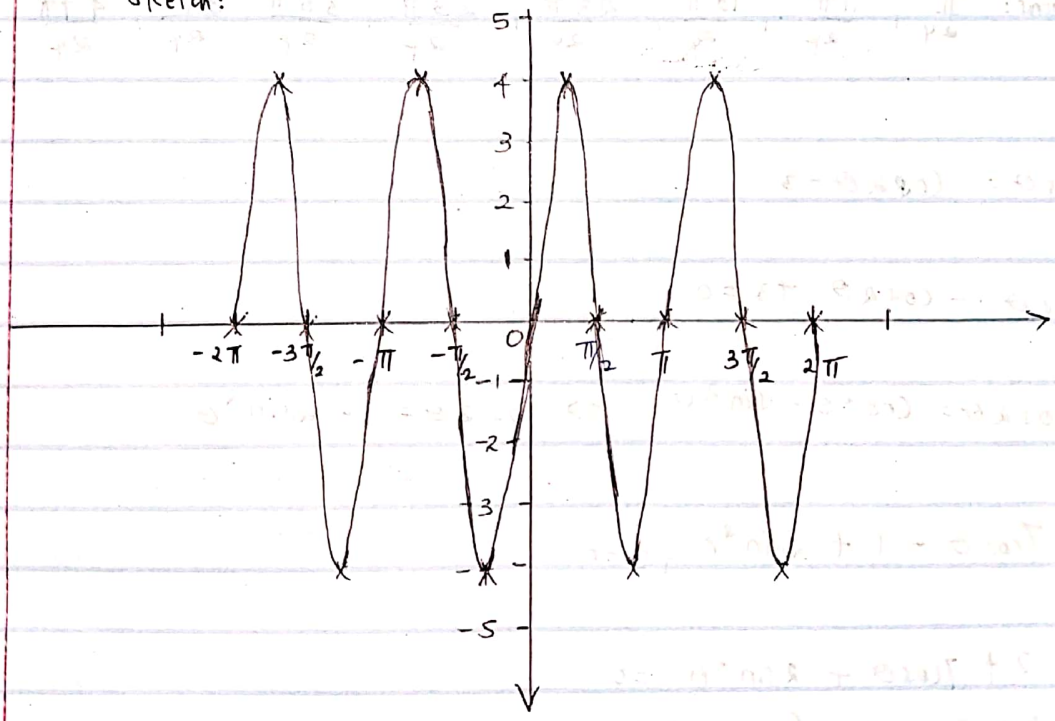
$4\sin 2x = 8 \sin x \cos x$

$= 4\sin 2x$

Amplitude: 4

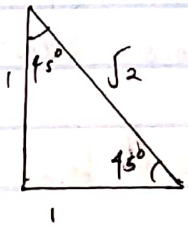
Period: $\frac{2\pi}{2} = \pi$

Sketch:



11 $\csc\theta - \sqrt{2} = 0$

$\csc\theta = \sqrt{2}$



$\frac{1}{\sin\theta} = \sqrt{2}$; $\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

(b) $2\cos 4\theta = \sqrt{3}$

$\cos 4\theta = \frac{\sqrt{3}}{2}$; let $4\theta = x$

$\cos x = \frac{\sqrt{3}}{2}$; $x = \frac{\pi}{6}$, $x = \frac{11\pi}{6}$

$\theta = \frac{\pi}{24}$, $\theta = \frac{11\pi}{24}$, $\theta = \pi + \frac{\pi}{24}$, $\theta = \pi + \frac{11\pi}{24}$,

$\theta = \frac{\pi}{4} + \frac{\pi}{2}$, $\theta = \frac{11\pi}{24} + \frac{\pi}{2}$

solutions: $\frac{\pi}{24}$, $\frac{11\pi}{24}$, $\frac{13\pi}{24}$, $\frac{23\pi}{24}$, $\frac{25\pi}{24}$, $\frac{35\pi}{24}$, $\frac{37\pi}{24}$, $\frac{47\pi}{24}$

(c) $7\cos\theta = \cos 2\theta - 3$

$7\cos\theta - \cos 2\theta + 3 = 0$

$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad \Rightarrow \cos 2\theta = 1 - 2\sin^2\theta$

$= 7\cos\theta - 1 + 2\sin^2\theta + 3 = 0$

$= 2 + 7\cos\theta + 2\sin^2\theta = 0$

$2 + 7\cos\theta + 2(1 - \cos^2\theta) = 0$

$= 4 + 7\cos\theta - 2\cos^2\theta = 0$

Let $\cos\theta$ be x

$4 + 7x - 2x^2 = 0$

solving quadratically, $x = \frac{1}{2}$, $x = 4$

$\cos x = \frac{1}{2}$, $\cos x = 4$ (No soln)

$x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$