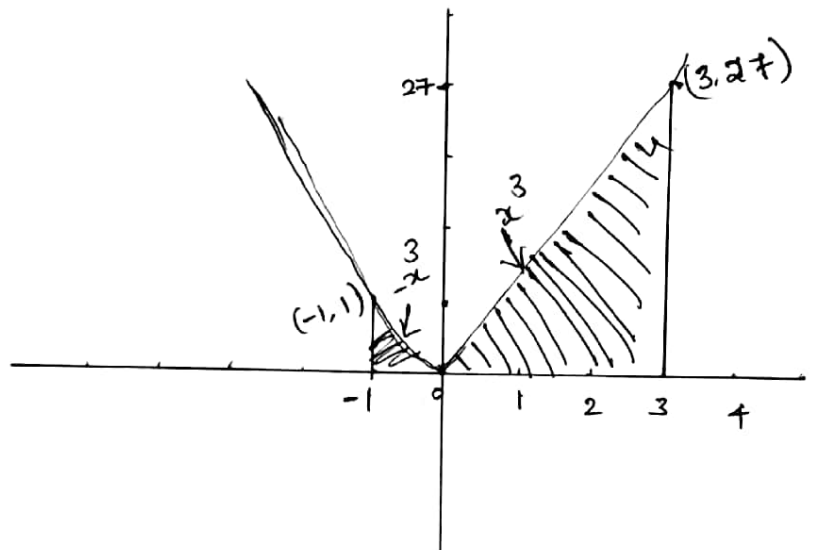


Question #7

a) If a function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$

b) Given function is $f(x) = |x^3|$, $x \in [-1, 3]$

$$f(x) = |x^3| = \begin{cases} x^3 & \text{if } x \geq 0 \\ -x^3 & \text{if } x \leq 0 \end{cases}$$



$$-x^3 \geq 0, \quad \forall x \in [-1, 0] \text{ and } x^3 \geq 0, \quad \forall x \in [0, 3]$$

$\therefore$ The area between the graph of $f(x)$ and the x-axis is

$$\begin{aligned} A &= \int_{-1}^0 f(x) dx + \int_0^3 f(x) dx \\ &= \int_{-1}^0 (-x^3) dx + \int_0^3 (x^3) dx \\ &= \left[-\frac{x^4}{4} \right]_{-1}^0 + \left[\frac{x^4}{4} \right]_0^3 \\ &= \frac{84}{4} \\ &= \frac{41}{2} \\ &= \underline{\underline{\frac{41}{2}}} \end{aligned}$$

Question #7

$$c) f(x) = \begin{cases} x^3 & \text{if } x \geq 0 \\ -x^3 & \text{if } x < 0 \end{cases}$$

Divide $[-1, 3]$ into 4 subintervals of equal length

$$\Delta x = \frac{3 - (-1)}{4} = \frac{3+1}{4} = 1, \text{ so that}$$

$$[-1, 3] = [-1, 0] \cup [0, 1] \cup [1, 2] \cup [2, 3]$$

Then by mid-point rule,

$$\begin{aligned} \int_{-1}^3 f(x) dx &= f\left(-\frac{1}{2}\right) \Delta x + f\left(\frac{1}{2}\right) \Delta x + f\left(\frac{3}{2}\right) \Delta x + f\left(\frac{5}{2}\right) \Delta x \\ &= -\left(-\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{5}{2}\right)^3 \quad \because \Delta x = 1 \\ &= -\left(-\frac{1}{8}\right) + \frac{1}{8} + \frac{27}{8} + \frac{125}{8} \\ &= \frac{1}{8} + \frac{1}{8} + \frac{27}{8} + \frac{125}{8} \\ &= \frac{154}{8} \\ &= \frac{77}{4} \end{aligned}$$

The error is

$$e = | \text{Exact value} - \text{Approximate value} |$$

$$e = \left| \frac{41}{2} - \frac{77}{4} \right| = \left| \frac{82}{4} - \frac{77}{4} \right| = \left| \frac{5}{4} \right| = \frac{5}{4}$$

$$\therefore e = \frac{5}{4} \approx \underline{\underline{1.25}}$$