

Question #8

- a) At time $t=0$, the position of a body moving along the s -axis is $s=0$. Its initial velocity is $v(0)=-10$.

The body is experiencing an acceleration of $a(t) = 6t - 6$.

Find the velocity $v(t)$ as follows:

$$v(t) = \int a(t) dt = \int (6t - 6) dt = \frac{6t^2}{2} - 6t + C = 3t^2 - 6t + C$$

Substitute the initial condition:

$$v(0) = -10$$

$$-10 = 3t^2 - 6t + C$$

$$-10 = 3(0)^2 - 6(0) + C$$

$$-10 = C$$

Hence, the velocity is $v(t) = 3t^2 - 6t - 10$.

b) ~~Find the position $s(t)$~~ $s(t) = \int v(t) dt$
 ~~$s(t) = \int (3t^2 - 6t - 10) dt$~~
 ~~$s(t) = t^3 - 3t^2 - 10t + C$~~
 ~~$s(0) = 0 \Rightarrow 0^3 - 3(0)^2 - 10(0) + C = 0 \Rightarrow C = 0$~~

c) Average velocity $v(t) = \frac{\int_2^4 dv dt}{4-2}$
 $= \frac{\int_2^4 (3t^2 - 6t - 10) dt}{2}$
 $= \frac{[t^3 - 3t^2 - 10t]_2^4}{2}$
 $= \underline{\underline{0}}$

d) $f'(c) = \frac{f(b) - f(a)}{b-a}$

In this case, we have $f'(c) = 0$. The special case, when $f(a) = f(b)$ is known as Rolle's Theorem. In other words, there exists a point in the interval (a, b) which has a horizontal tangent. So instantaneous velocity $v(t)$ must have been equal to zero at some point in the time interval $[2, 4]$.

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$$\begin{aligned}
 \text{e) } V_{\text{avg}} &= \frac{1}{4-2} \int_2^4 (6t^2 - 6t - 10) dt \\
 &= \frac{1}{2} \int_2^4 (6t^2 - 6t - 10) dt \\
 &= \frac{1}{2} \left(\int_2^4 6t^2 dt - 6 \int_2^4 t dt - 10 \int_2^4 dt \right) \\
 &= \frac{1}{2} [112 - 36 - 20] \\
 &= \frac{1}{2} \times 56 \\
 &= \underline{\underline{28}}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } \Delta V_{(a,b)} &= \frac{s(b) - s(a)}{b-a} \\
 &= \frac{(6 \times 4 - 6) - (6 \times 2 - 6)}{4-2} \\
 &= \underline{\underline{54}}
 \end{aligned}$$

g)

