

Midterm 1 Samples

Sample 1

1. Prove or disprove: $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are equivalent.
2. Let $P(x, y)$ be “ x and y are real numbers such that $x + 2y = 5$ ”. What are the truth values of $\forall x \exists y P(x, y)$ and $\exists x \forall y P(x, y)$ and $\forall y \exists x (x \leq y \wedge P(x, y))$?
3. Prove that $x^4 + y^3 = 17$ has a unique positive integer solution.
4. Let $F(A)$ be the predicate A is a finite set and $S(A, B)$ be the predicate A is contained in B . Suppose the universe of discourse consists of all sets. Translate each statement into symbols.
 - (a) No infinite set is contained in a finite set.
 - (b) The empty set is a subset of every finite set.
5. Let A , B , and C be sets. Prove or disprove that $A - (B \cap C) = (A - B) \cup (A - C)$.
6. Give an example of a function $f : \mathbb{N} \rightarrow \mathbb{Z}$ that is both one-to-one and onto.

Sample 2

1. Determine whether this proposition is a tautology: $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$.
2. Suppose $P(x, y)$ is a predicate and the universe for the variables x and y is $\{1, 2, 3\}$. Suppose $P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2)$ are true, and $P(x, y)$ is false otherwise. What are the truth values of $\forall x \exists y P(x, y)$ and $\exists x \forall y P(x, y)$ and $\forall y \exists x (x \leq y \wedge P(x, y))$?
3. Prove that $n^3 + 1$ is odd if and only if $n^2 - 1$ is odd.
4. Write the negation of the following propositions.
 - (a) Some students in my class use e-mail.
 - (b) If xy is irrational then either x is irrational or y is irrational.
5. Consider the function $f(n) = 2\lfloor n/2 \rfloor$ from $\mathbb{Z}$ to $\mathbb{Z}$. Is this function one-to-one? Is this function onto? Justify your answer.
6. Find $\bigcap_{i=1}^{\infty} (i, \infty)$ and $\bigcup_{i=1}^{\infty} (i, \infty)$.
7. Suppose $A = \{a, \{b\}\}$. Is it true that $\{\{\{b\}\}\} \subseteq \mathcal{P}(A)$? Is $\{b\} \in \mathcal{P}(A)$?