

Question # 1.

$$y = f(x) = 5x + 7 - 18e^x$$

$$m = f'(x) \Big|_{P(x,y)}$$

$$y_0 = -11$$

$$f'(x) = 5 - 18e^x$$

$$m = f'(x) \Big|_{(0,-10)} = 5 - 18e^0 = -13$$

Now for y-intercept, 'b'

equation of line

$$y - y_0 = m(x - x_0)$$

$$y + 11 = -13x$$

$$y = -13x - 11$$

comparing with

$$y = mx + b$$

$$m = -13$$

$$b = -11$$

Q # 2.

(a) $f(x) = (\ln x)^7$

$$f'(x) = \frac{d}{dx} (\ln x)^7$$

Using power rule and
derivative of $\ln x$

$$f'(x) = 7 (\ln x)^{7-1} \cdot \frac{d}{dx} \ln x$$

$$= 7 (\ln x)^6 \cdot \frac{1}{x}$$

$$= \frac{7}{x} \cdot (\ln x)^6$$

(b)

Here

$$f'(e^x) = \frac{7}{e^x} \cdot (\ln e^x)^6$$

$$= \frac{7}{e^x} \cdot x^6 \Rightarrow 7x^6 e^{-x}$$

$$\boxed{f'(e^x) = 7x^6 \cdot e^{-x}}$$

Q# 3

$$g(x) = (6x^3 - 4x + 1)e^x$$

Use
Differentiating w.r.t x

$$g'(x) = \frac{d}{dx} ((6x^3 - 4x + 1)e^x)$$

using Product rule.

$$= \frac{d}{dx} e^x \cdot (6x^3 - 4x + 1) +$$

$$e^x \cdot \frac{d}{dx} (6x^3 - 4x + 1)$$

$$= e^x (6x^3 - 4x + 1) + e^x (18x^2 - 4)$$

$$= e^x (6x^3 - 4x + 1 - 18x^2 - 4)$$

$$\boxed{g'(x) = e^x (6x^3 - 18x^2 - 4x - 3)}$$

Question 4.

$$R = 1000x - x^2$$

$$\begin{aligned} \text{Rate of increase of sales} &= 50 \text{ units} \\ \text{Number of units sold} &= 950 \end{aligned}$$

Per day

Here Revenue is changing
with time

Differentiating R w.r.t 't'

$$\begin{aligned} \frac{d}{dt}(R) &= \frac{d}{dt}(1000x - x^2) \\ &= (1000 - 2x) \frac{dx}{dt} \end{aligned}$$

Now, Sales are increasing at rate of
50 units per day

$$\frac{dx}{dt} = 50$$

$$\text{where } x = 950$$

So

$$\frac{dR}{dt} = (1000 - 1900)(50)$$

$$\boxed{\frac{dR}{dt} = 45,000 \text{ dollars per day}}$$

Q # 5

$$43x^2 - xy + 2y^3 = -60$$

Differentiating w.r.t x

$$\frac{d}{dx}(43x^2) - \frac{d}{dx}(xy) + 2\frac{d}{dx}(y^3) = \frac{d}{dx}(-60)$$

$$86x - (y + x\frac{dy}{dx}) + 6y^2\frac{dy}{dx} = 0$$

$$86x - y - x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(6y^2 - x) = -86x + y$$

$$\frac{dy}{dx} = \frac{(-86x + y)}{(6y^2 - x)}$$

Now

$$\text{slope} = \frac{dy}{dx} \bigg|_{(-1,1)} = \frac{86+1}{6+1} = \frac{-87}{7}$$

$$\text{Slope} = \frac{87}{7}$$

Q # 6

$$x = \frac{9000}{\sqrt{5p+1}}$$

$$R = xp = \frac{9000p}{\sqrt{5p+1}}$$

$$\frac{dR}{dp} = \frac{\sqrt{5p+1}(9000) - 9000p \cdot \frac{5}{2} \cdot (5p+1)^{-1/2}}{5p+1}$$

$$= \frac{9000(5p+1) - 22500p}{(5p+1)^{3/2}}$$

$$\frac{dR}{dp} = \frac{9000 + 22500p}{(5p+1)^{3/2}}$$

$$p = 250, \quad \frac{dp}{dt} = 2$$

$$\frac{dR}{dp} = 127.33$$

$$\boxed{\frac{dR}{dp} = 127.33}$$

Now

$$\frac{dR}{dt} = \frac{dR}{dp} \times \frac{dp}{dt} = (127.33 \times 2)$$

$$\boxed{\frac{dR}{dt} = 254.66}$$

Q#7

$$f(x) = (4x-8) e^{4x}$$

Differentiating w.r.t x

$$\begin{aligned} f'(x) &= \left[\frac{d}{dx} e^{4x} \right] (4x-8) + \left[\frac{d}{dx} (4x-8) \right] e^{4x} \\ &= 4e^{4x} (4x-8) + 4 \cdot e^{4x} \\ &= 4e^{4x} (4x-8+1) \end{aligned}$$

$$\boxed{f'(x) = 4e^{4x} (4x-7)}$$

For critical value 'c'

$$f'(x) = 0.$$

$$4e^{4x} (4x-7) = 0.$$

$$4x = 7 \Rightarrow$$

$$\boxed{x = \frac{7}{4}}$$

Here

$$\boxed{c = \frac{7}{4}}$$

is critical value.

Q# 8

$$(a) \quad x^{1/4} y^{3/4} = 320000$$

Differentiating w.r. to x

$$\frac{d}{dx} (x^{1/4} \cdot y^{3/4}) = \frac{d}{dx} (320000)$$

$$x^{1/4} \cdot \frac{3}{4} y^{-1/4} \frac{dy}{dx} + y^{3/4} \cdot \frac{1}{4} x^{-3/4} = 0$$

$$\frac{3}{4} x^{1/4} y^{-1/4} \frac{dy}{dx} = -\frac{1}{4} x^{-3/4} y^{3/4}$$

$$\frac{dy}{dx} = \frac{-3 \cdot x^{-3/4} y^{3/4}}{x^{1/4} y^{-1/4}}$$

$$\frac{dy}{dx} = -3 \frac{y}{x} = \frac{-3y}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{-3y}{x}}$$

$$(b) \quad \left. \frac{dy}{dx} \right|_{(144, 576)} = \frac{-3(576)}{144} = -12$$

Hence investment decrease by
12 dollars.