

SOLUTIONS

Consider the one-sample t-test with a normal population $N(\mu, \sigma^2)$;

$$H_0: \mu = 0, \text{ versus } H_1: \mu > 0.$$

(a) (i) Test statistic of one-sample t-test.

solution

$$T = \frac{\bar{x}_i - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

where, $\bar{x}_i$ - sample mean

μ_0 - population mean (hypothesized)

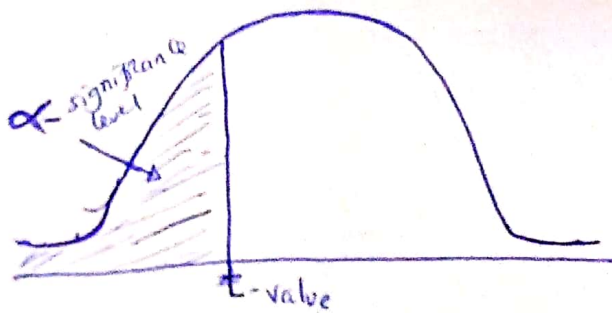
s - sample std. dev

n - sample size

(ii) When H_0 is true:

The sampling distribution is a normal distribution.

(iii) Sketch to illustrate calculation of one-sided p-value:



NB

If p-value is greater than α -significance level, we fail to reject H_0 , otherwise we reject.

(b) Solution

Cumulative distr. function $F(x)$ is defined as $F(x) = P(X \leq x)$
with the following relationship between c.d.f and p.d.f.

$$F(x) = \int_{-\infty}^x f_x(u) du,$$

$\Rightarrow$ The p.d.f will be a non-negative Lebesgue integrable function such that:

$$P_r[a \leq X \leq b] = \int_a^b f_x(x) dx.$$

for strictly and continuous, the p.d.f will be:

$$= \int_a^{\infty} f_x(x) dx$$

(c) Solution

After the simulation, and assuming significance level $\alpha = 0.05$,
the p-value is not the probability of H_0 being true
is that the p-value is less than α (significance level)

(d) Solution

Main differences between p-values in A and B-D.

- The p-values are decreasing from B-D while those of histogram A are approximately constant.

Reason: This is because the mean = 0 for A makes the ~~test~~ standardized test statistics constant but the ones for B-D are varying.

(e) Among A-D, which one has highest chance of observing a p-value less than $\alpha = 0.01$?

Solution

- Histogram A.

- Because it has a constant mean of zero (0), which implies that the test statistics will be zero.