

10(1)

WEDSDAY [13]

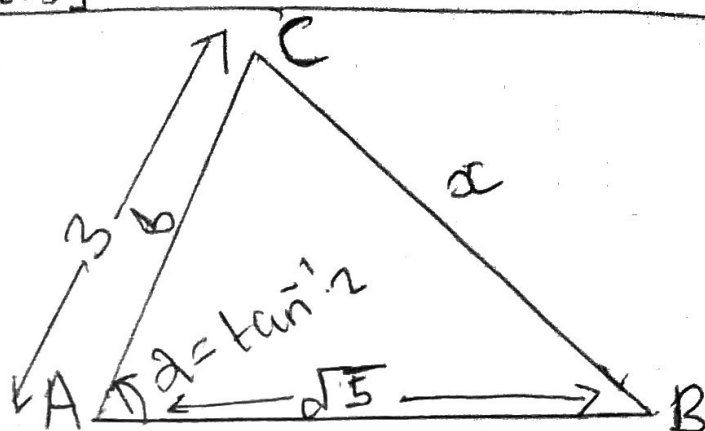
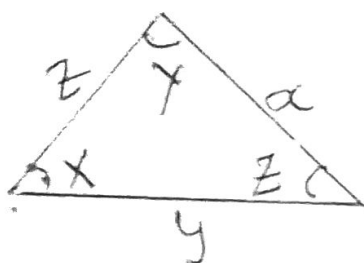
GIVEN $\triangle ABC$

$$AB = a = \sqrt{5}$$

$$AC = b = 3$$

$$BC = x$$

$$\angle BAC = \alpha = \tan^{-1} 2$$

Find x (as a simplified algebraic number) $\cdot a$ USE COSINE RULE

$$y^2 = x^2 + z^2 - 2xz \cos Y$$

$$z^2 = x^2 + y^2 - 2xy \cos Z$$

$$x^2 = y^2 + z^2 - 2yz \cos X$$

To get x ;

$$x^2 = 3^2 + (\sqrt{5})^2 - 2(3 \times \sqrt{5}) \cos 63.43^\circ$$

$$x^2 = 9 + 5 - 6\sqrt{5} \cos 63.43^\circ \quad \Rightarrow \tan^{-1} 2 = 63.43^\circ$$

$$x^2 = 14 - 6\sqrt{5} \cos 63.43^\circ$$

$$x^2 = 14 - 6.0010$$

$$x^2 = 7.9990$$

$$x = \sqrt{7.9990}$$

$$= 2.8282 \text{ units}$$

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GIVEN $\frac{e^x}{e^x - 3} = 2$ FIND,

(a) The set, Λ , so that

$$\frac{e^x}{e^x - 3} = 2 \Leftrightarrow x \in \Lambda$$

(b) Count the number of elements in Λ , $n(\Lambda)$.

Solution,

$$\frac{e^x}{e^x - 3} = 2 \quad \text{Multiply both sides by } e^x - 3 \text{ to get}$$

$$e^x = 2(e^x - 3) \quad \text{- open the brackets}$$

$$e^x = 2e^x - 6 \quad \text{- Put the like terms together}$$

$$2e^x - e^x = 6$$

$$e^x = 6 \quad \text{Take Natural logs on both sides}$$

$$\ln(e^x) = \ln 6$$

$$x \ln e = \ln 6 \quad \text{but } \ln e = 1$$

$$x = \ln 6 = 1.791759469$$

(b) Number of elements in Λ is only ①

$$n = 1$$

PROVE THE IDENTITY $\frac{\cos x}{1 - \sin x} = \sec x + \tan x$

Soln

✓ $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = (1 - \sin x)(1 + \sin x)$

Difference of two squares

✓ $\frac{\sin x}{\cos x} = \tan x$

✓ $\sec x = \frac{1}{\cos x}$

So $\frac{\cos x}{1 - \sin x} = \sec x + \tan x$

$\frac{\cos x (1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$

Multiply both numerator and denominator by $(1 + \sin x) \Rightarrow$ This is unnecessary one

$\frac{(\cos x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$

$\Rightarrow \text{Recall } (1 - \sin x)(1 + \sin x) = 1 - \sin^2 x = \cos^2 x$

$\frac{(\cos x)(1 + \sin x)}{\cos^2 x}$

$\cos^2 x = (1 - \sin x)(1 + \sin x) = 1 - \sin^2 x$

$\frac{\cancel{\cos x} (1 + \sin x)}{\cancel{\cos x} \times \cos x} = \frac{1 + \sin x}{\cos x}$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} \Rightarrow \text{As shown above}$

$= \sec x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \tan x$

$= \sec x + \tan x$ as required to prove

15(4)

GIVEN $\cdot a_k = 2k + 1$

EVALUATE : $\sum_{k=1}^4 a_k$

Solution

$k = 1, 2, 3, 4$

$$\sum_{k=1}^4 a_k = [2(1) + 1] + [2(2) + 1] + [2(3) + 1] + [2(4) + 1]$$

$$= [2 + 1] + [4 + 1] + [6 + 1] + [8 + 1]$$

$$= [3] + [5] + [7] + [9]$$

$$= 8 + 16$$

$$= 24$$