Exam 2 Ikhlas Suleyman

Question 1

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Question 2

(a) $\frac{d}{dx}(\tan x) = \sec^2 x$

$\tan x = \frac{\sin x}{\cos x}$

$\frac{d}{dx}\left(\frac{\sin x}{\cos(x)}\right) = \frac{\cos(x)\, d/dx(\sin x) - \sin(x)\, d/dx(\cos(x))}{\cos^2 x}$

$= \frac{\cos^2 x - (\sin x \cdot -\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

but $\cos^2 x + \sin^2 x = 1$

$= \frac{1}{\cos^2 x} = \sec^2 x$

(b) $\frac{d}{dx}\left(\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}}$

$y = \cos^{-1}(x)$

$y' = [\cos^{-1}(x)]'$

$\cos y = x \rightarrow [\cos y]' = x'$

$y'(-\sin y) = 1$

$-\sin(\cos^{-1}(x)) \cdot y' = 1$

$y' = \frac{-1}{\sin(\cos^{-1}(x))}$; but $\sin\alpha = \sqrt{1-\cos^2\alpha}$

$= \frac{-1}{\sqrt{1-\cos^2(\cos^{-1}(x)}}$; $\cos(\cos^{-1}(x)) = x$

$= \frac{-1}{\sqrt{1-x^2}}$

Thus $\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

<u>Question 3</u>

$y = \cot^6\left(\sqrt[3]{\arcsin(9x)}\right)$

$= \left[\cot\left(\sqrt[3]{\arcsin(9x)}\right]^6\right.$

$$= 6\cot^5\left(\sqrt{\arcsin(9x)}\right) \times \frac{d}{dx}\cot\left(\sqrt[3]{\arcsin(9x)}\right)$$

$$= 6\cot^5\left(\sqrt[3]{\arcsin(9x)}\right)\left(-\csc^2\left(\sqrt[3]{\arcsin(9x)}\right)\right) \cdot d/_{dx}\left(\sqrt[3]{\arcsin(9x)}\right)$$

$$= -6\cot^5\left(\sqrt[3]{\arcsin(9x)}\right) \tfrac{1}{3}\arcsin^{-2/3}(9x) \times d/_{dx}\arcsin 9x \times \csc^2\sqrt[3]{\arcsin 9x}$$

$$= \frac{2\cot^5\left(\sqrt[3]{\arcsin(9x)}\right) \cdot \frac{1}{\sqrt{1-9x^2}} \cdot d/_{dx}\, 9x \cdot \csc^2\left(\sqrt[3]{\arcsin(9x)}\right)}{\arcsin^{2/3}(9x)}$$

(b) $y = \dfrac{8^{\pi x}\ln(x)}{\operatorname{arccot}(4x)}$

Applying Quotient rule

$\dfrac{d}{dx}\operatorname{arccot}(4x) : \dfrac{-1}{4x^2+1} \times d/_{dx}\, 4x$ ie $(\operatorname{arccot}(u))' = \dfrac{1}{4(x)^2+1}$

$$= \frac{-4}{16x^2+1}$$

$$\frac{d}{dx} 8^{\pi x}\ln x = \ln(8)\cdot 8^{\pi x} \times d/_{dx}(\pi x) \times \ln(x) + 8^{\pi x}\cdot\frac{1}{x}$$

$$= \pi\ln(8)\cdot 8^{\pi x}\ln(x) + \frac{8^{\pi x}}{x}$$

$$= \frac{8^{\pi x}(\pi\ln(8)\, x\ln(x) + 1)}{x}$$

$$y' = \frac{\dfrac{\operatorname{arccot}(4x)\left(8^{\pi x}[\pi\ln(8)\, x\ln(x)+1]\right)}{x} + \dfrac{8^{\pi}\ln(x)\cdot 4}{16x^2+1}}{(\operatorname{arccot}(4x))^2}$$

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$$= \frac{\operatorname{arccot}(4x)\left(3\pi \ln 2 \cdot 8^{\pi x} \ln(x) + \frac{8^{\pi x}}{x}\right)(16x^2+1) + 2^{2+3\pi x}\ln(x)}{\operatorname{arccot}^2(4x)\ (16x^2+1)}$$

(i) $y = (x-2)^{x+1}$

$$\ln(y) = x+1 \ln(x-2)$$

$$\frac{d}{dx} \ln(y) = \frac{d}{dx}\left((x+1)\ln(x-2)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x+1}{x-2} + x\ln(x-2)$$

$$\frac{dy}{dx} = y\left(\frac{x+1}{x-2} + x\ln(x-2)\right)$$

$$= (x-2)^{x+1}\left(\frac{x+1}{x-2} + x\ln(x-2)\right)$$

Question 4

$y^2 + \ln(xy) = 2$

find y'

$$\frac{d}{dx} y^2 + \frac{d}{dx}\ln(xy) = \frac{d}{dx}(2)$$

$$\frac{1}{yx}\frac{d}{dx}(yx) + 2y\frac{dy}{dx} = 0$$

$$\frac{1}{yx}\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$

$$\frac{x}{yx}\frac{dy}{dx} + \frac{y}{yx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(\frac{x}{yx} + 2y\right) = \frac{-1}{x}$$

$$\frac{dy}{dx} = \frac{-1}{x\left(x/y + 2y\right)} = \frac{-1}{\frac{x^2}{y} + 2yx}$$

$$y' = \frac{-1}{2xy^2 + x}$$

(b) tangent @ $(e, 1)$

$$y - 1 = m(x - e) \quad ; \quad m = \frac{-1}{2e + e} = \frac{-1}{3e}$$

$$y - 1 = \frac{-1}{3e}(x - e)$$

$$= -x/3e + \frac{e}{3e} = \frac{-x}{3e} + 1/3$$

$$y = -x/3e + 4/3$$

Question 5

$$f(x) = \frac{x+3}{x+1}$$

$$\frac{x+3}{x+1} = 2$$

$$2x + 2 = x + 3 \quad ; \; x = 1$$

$$(f^{-1})'(2) = \frac{1}{f'(1)} \quad ; \; f'(x) = \frac{(x+1) - (x+3)}{(x+1)^2}$$

$$\frac{1}{f'(x)} = \frac{-(x+1)^2}{2} \quad ; \; \frac{1}{f'(1)} = \frac{-(1+1)^2}{2}$$

$$= -4/2 = -2$$

$$(f^{-1})'(2) = -2$$

Question 6

(a) $f'(x)$ DNE at $x = -1$ and $x = 2$. Graph has a corner

(b) Local minimum : $(2, -2)$ and $(6, 1)$

local maximum : $(-1, 1)$ and $(4, 2)$

(c) Absolute minimum: $(2, -2)$

Absolute maximum : $(3, 1)$

Question 7

$f(x) = x\sqrt{1-x^2} = x(1-x^2)^{1/2}$

$f'(x) = x\left(\frac{1}{2}\cdot(1-x^2)^{-1/2}\cdot 2x\right) + \sqrt{1-x^2}$

$f'(x) = \dfrac{-2x^2+1}{\sqrt{1-x^2}} = 0$

$2x^2 = 1$

$x^2 = 1/2$; $x = \sqrt{1/2}$; $x = -\sqrt{1/2}$

$f(\sqrt{1/2}) = \sqrt{0.5}\sqrt{1-1/2} = 1/2$; $f(-\sqrt{1/2}) = -1/2$

∴ Absolute Maximum : $(\sqrt{1/2}, 1/2)$

Absolute minimum : $(-\sqrt{1/2}, -1/2)$

Question 8

$C(x) = 10\left(\frac{1}{x} + \frac{x}{x+3}\right)$

(a) $C(3) = 10\left(1/3 + 3/6\right) = 25/3$

$C(6) = 10\left(1/6 + \frac{6}{9}\right) = 25/3$

hence verified.

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$$C'(x) = 10 \frac{d}{dx}\left(\frac{1}{x} + \frac{x}{x+3}\right)$$

$$= 10\left(-\frac{1}{x^2} + \frac{(x+3) - x}{(x+3)^2}\right)$$

$$= 10\left(-\frac{1}{x^2} + \frac{3}{(x+3)^2}\right)$$

$$\frac{-1}{x^2} + \frac{3}{(x+3)^2} = 0$$

$$\frac{-(x+3)^2 + 3x^2}{x^2(x+3)^2} = 0$$

$$-(x+3)^2 + 3x^2 = 0$$

$$-(x^2+9+2x)+3x^2 = 0$$

$$2x^2 - 9 + 2x = 0$$

$$x = \frac{3(1+\sqrt{3})}{2} \; ; \; x = \frac{3(1-\sqrt{3})}{2}$$

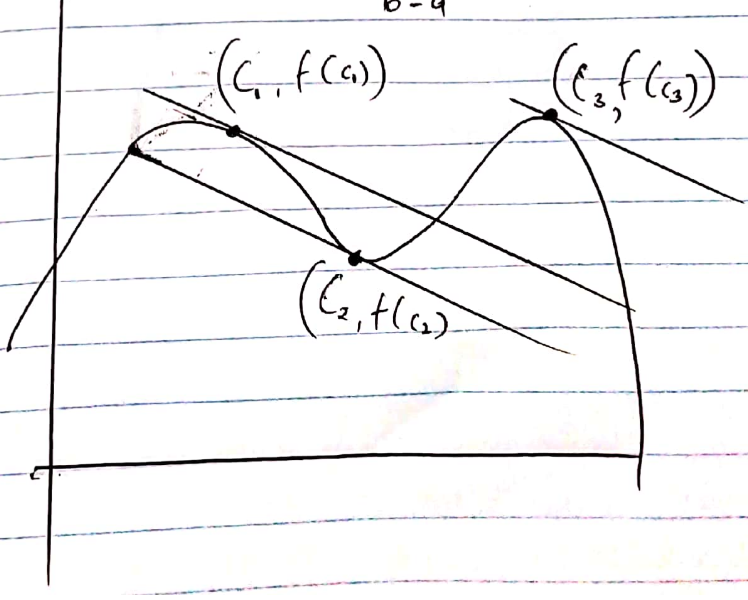
Order size = 4 components ; $x = \frac{3(1 \pm \sqrt{3})}{2} \approx 4$ (tue)

Question 9

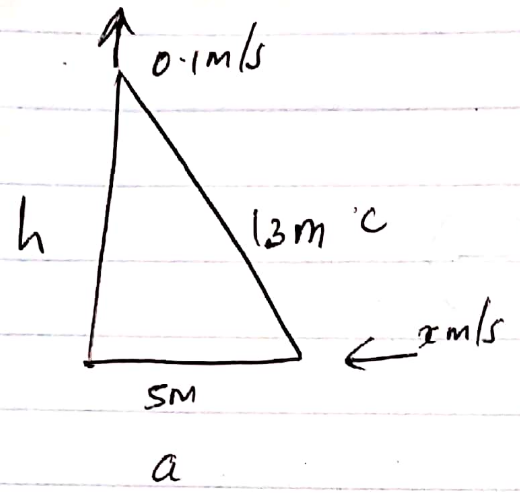
$f(-1) = f(1) = 2$

$$f'(x) = \frac{2x}{\sqrt{1-(x^2-1)^2}}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



Question 10



$h = \sqrt{13^2 - 25}$

$= 12$

$\frac{dh}{dt} = 0.1$

$c^2 = a^2 + h^2$

$\frac{da}{dt} = -x \text{ m/s}$

$\frac{dc}{dt} = 0$

$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2h\frac{dh}{dt}$

$0 = 2 \cdot 5 \cdot -x + 2 \cdot 12 \cdot 0.1$

$0 = -10x + 2.4$

$10x = 2.4$

$x = -0.24 \text{ m/s}$

$= -0.24 \text{ m/s}$

Question 11

$$\sin(x) = x^2 - 3x + 1$$

$$x_1 = x_0 - \frac{-x_0^2 \cdot 3x_0 + \sin x_0) - 1}{-2x_0 + \cos(x_0) + 3}$$

$$= 0 - (0.25)$$

$$= 0.25$$

$$x_2 = 0.25 - (-0.018765)$$

$$= 0.26877$$

$$x_3 = 0.26877 - (-0.000116)$$

$$= 0.268881$$

$$x_p = 0.268881 - (-4.4329 \times 10^{-9})$$

$$= 0.26881$$

(a)